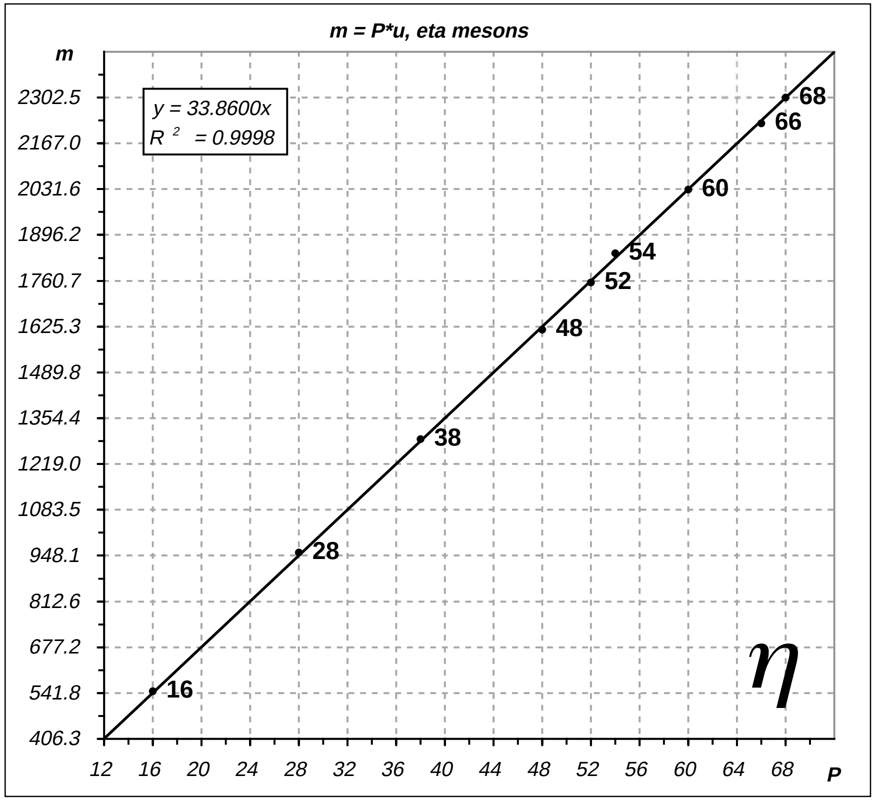

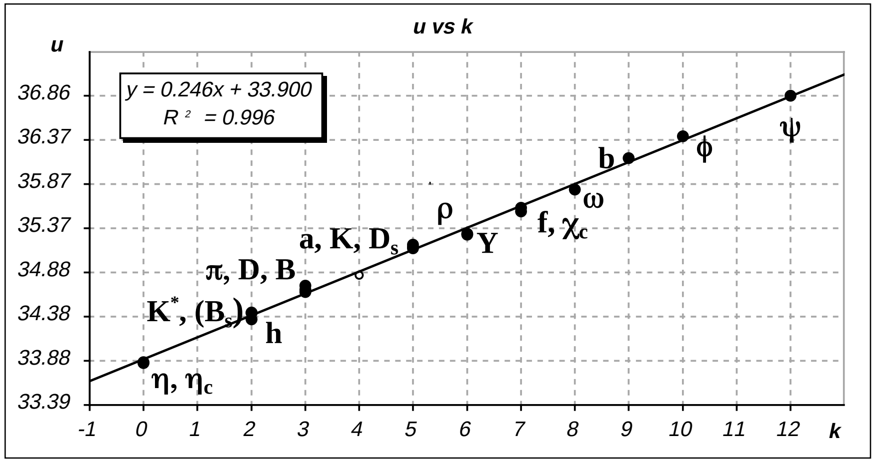

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28-JUL-2004
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Patterns in the Meson Mass Spectrum

Paolo Palazzi

Abstract

The conjecture that particle masses are multiples of a unit u of about 35 MeV has been proposed in various forms by several authors: mesons are even multiples of u, leptons and baryons odd multiples. Here this mass quantization is reassessed for all particles with mass below 1 GeV (stable leptons and $f_0(600)$ excluded), and found to be statistically significant. Subsequently all the mesons listed by the PDG are grouped in families defined by quark composition and J^{PC}, and analyzed for even mass multiplicity with a unit close to 35 MeV separately for each group. For all the the families that can be analyzed unambiguously this multiplicity hypothesis is found to be statistically significant. Most scalar and vector families show a dependence of u from the spin, while for pseudoscalars the effect is not present. Only 5 states out of 120 are rejected due to abnormally large fit residuals. The mass units of the various families are quantized on a grid of 12 intervals of about 0.25 MeV, ranging from 33.88 up to 36.86 MeV. The location of the values on the u-grid shows an intriguing pattern of correlation with the quantum numbers.

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Table of Contents

Revision Record

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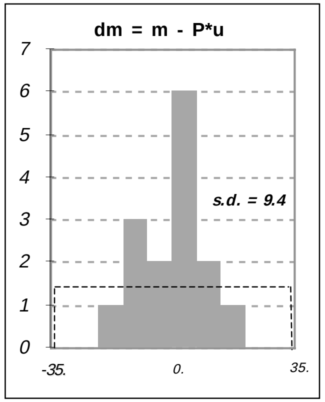
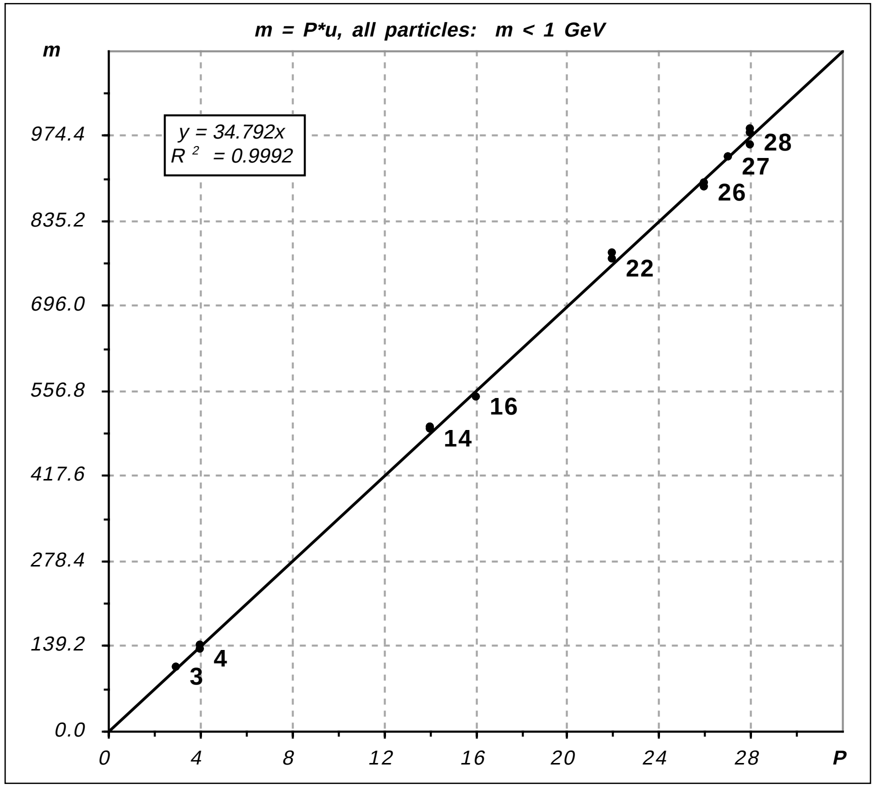
05-APR-2005 revision1:
abstract modified,
weighted fit paragraph in section 3.2 (page 9) modified;
missing rho point in figure 25 added, kaon point moved to k=5;
conclusions rephrased;
scrambled references to sections of appendix A fixed.

particle	m, MeV	errm	P	P*u	m-P*u	dm/m %	m/P
μ	105.66	5.0E-06	3	104.38	1.28	1.21	35.219
π^+	139.57	3.5E-04	4	139.17	0.40	0.29	34.893
π^0	134.98	6.0E-04	4	139.17	-4.19	-3.10	33.744
K^+	493.68	1.6E-02	14	487.08	6.59	1.34	35.263
K^0	497.67	3.1E-02	14	487.08	10.59	2.13	35.548
η	547.30	1.2E-01	**16**	556.67	-9.37	-1.71	**34.206**
ρ	771.10	9.0E-01	22	765.42	5.68	0.74	35.050
ω	782.57	1.2E-01	22	765.42	17.15	2.19	35.571
K^{*+}	891.66	2.6E-01	26	904.58	-12.92	-1.45	34.295
K^{*0}	896.10	2.7E-01	26	904.58	-8.48	-0.95	34.465
p	938.27	4.0E-05	27	939.38	-1.10	-0.12	34.751
n	939.57	4.0E-05	27	939.38	0.19	0.02	34.799
η'	957.78	1.4E-01	**28**	974.17	-16.39	-1.71	**34.206**
$f_0(980)$	980.00	1.0E+01	28	974.17	5.83	0.60	35.000
$a_0(980)$	984.70	1.2E+00	28	974.17	10.53	1.07	35.168
standard deviation --->					9.41	1.54	0.533

1a



1c



1b

Fig. 1. Mass unit: 1a, 35 MeV-based multiplicity assignment P for all particles with mass below 1 GeV; 1b, m vs P, line fit, ad-hoc mass scale; 1c, histogram of residuals dm = m-P*u, compared with a flat distribution in the (-34.8, 34.8) range.

1. The 35 MeV mass unit

From the early days of particle physics a number of authors have reported evidence of a mass quantization based in some form or another on a mass unit of about 35 MeV. This result, although statistically significant, has largely been ignored.

1.1 History

In 1952 Y. Nambu observed that the masses of the few particles known at that time were multiple (bosons) or half-multiple (fermions) of a mass unit u = 137 electron masses, about 70 MeV [1]. His sample included the muon, the pion, the proton and the K, thereby considering leptons, mesons and baryons together. Nambu noticed also that the mass of the electron as well as the proton-neutron and π^+ - π^0 mass differences could not be explained by the rule, and might correspond to a kind of fine structure.

In 1970 M. H. Mac Gregor referred to a mass quantum of 70 MeV, and expressed the masses of several mesons as integer multiples of this quantity [2]. In 1980 he observed that a few dozen mass differences among mesons and among baryons of the same kind were multiples of 140 MeV, and that this applied also to the τ-μ mass difference [3]. In the same year E. Jensen expressed the masses of the first four SU(3) meson and baryon multiplets as multiples of 1/4 the mass of the π^+, with a very detailed statistical analysis but no physical hypothesis [4].

In 1990 Mac Gregor proposed a constituent-quark (CQ) model, an attempt to combine the 35 MeV mass multiplicity with the quark model [5]. D. Akers in 1994 related quark masses and excitation quanta of the CQ model to Nambu's mass unit [6] and later developed extensions of the CQ model. Recently S. Giani noticed two remarkable quasi-identity mass formulae [7], and observed that, by replacing the masses with the integer multiplicity corresponding to a unit of 35 MeV, both equations turned into identities.

1.2 Mass unit for particles with mass < 1 GeV

The goal of the present paper is to assess the 35 MeV mass multiplicity hypothesis on (almost) all particles with mass < 1 GeV, and extend the analysis to all known mesons.

Figure 1a is the table of integer multiplicities P_i : $\mathbf{m_i = P_i{*}u}$ for all particles with mass below 1 GeV (photon, stable leptons and $f_0(600)$ excluded), using u = 34.79 MeV, the result of the linear least squares (LS) fit of figure 1b, with R^2 correlation coefficient $R^2(e)$= 0.9992 (please refer to appendix A.1 for the definition of R^2).

Is this correlation statistically relevant? Is it unique?

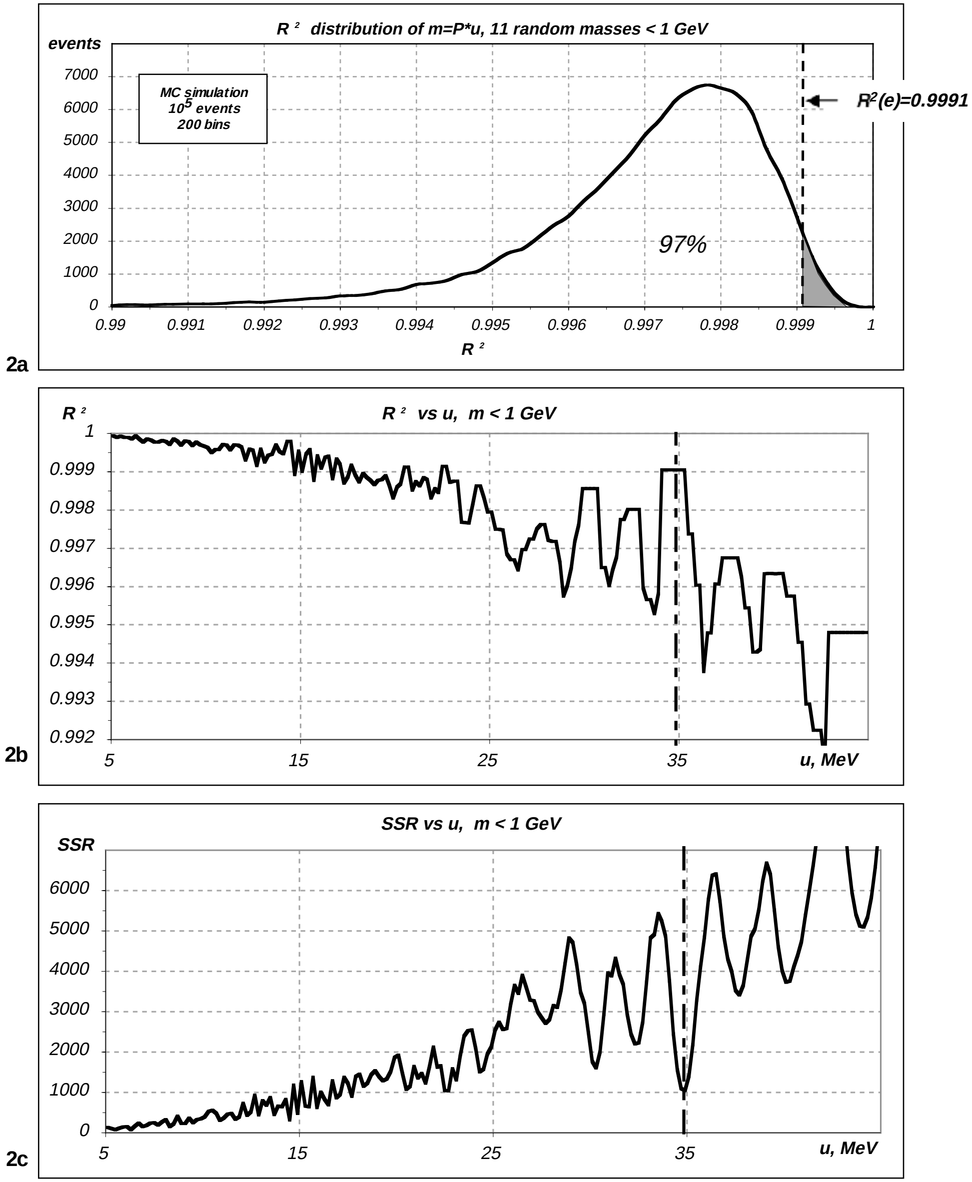


Fig. 2. Statistical relevance of the mass unit hypothesis: 2a, Montecarlo distribution of R^2 for 11 random masses below 1 GeV and estimate of the p-value; 2b, R^2 versus u for the 11 particles below 1 GeV ; 2c, Sum of squares of residuals versus u for the same sample.

2. Statistical Relevance

The $m_i = P_i*u$ hypothesis is approximate. The mass differences within isospin multiplets are not accounted for, the residuals $dm_i = m_i-P_i*u$ are generally sizable in comparison with the errors on the masses (errm in table 1a), the m/P ratio ranges from less than 34 up to 35.5 MeV, and there are particles with the same P and different u at P=22 and P=28. On the other hand the m/P ratio for the η and η' is the same to 5 significant digits, the η'/η mass ratio being equal to 1.750009 ~ 7/4 = 28/16 [7].

To handle the I-multiplet split, the sample is reduced from 15 to 11 by averaging the masses within the π, K, K* and N multiplets. The corresponding line fit yields u=34.83 with R^2(e) of 0.9991 (plot not shown). The P_i are computed from the masses using the mass unit, therefore a large fraction of the correlation between m and P is by construction.

Taking R^2 as goodness-of-fit statistics, with a Montecarlo simulation 11 random masses below 1 GeV are generated, and the corresponding P_i are computed using u = 34.83 and assuming that P is even except for 2 out of 11 particles where it is odd (the reduced physical sample consists of 9 mesons, the muon and the nucleon). This is repeated 10^5 times, then the R^2 value of the experimental sample R^2(e) is compared with the MC distribution: the fraction of the integral of the distribution between 0 and R^2(e) is 0.97 (figure 2a). Following the recommendations of the PDG, this estimator of the agreement between the data and the hypothesis is referred to as "p-value" rather than "confidence level", as explained in appendix A.4.

It might be that different values of the mass unit produce different sets of P_i also with an acceptable p-value. This eventuality can be probed with a u-scan, by varying u in small steps and analyzing the behavior of the R^2 of the (m_i ,P_i) distribution, where m_i are the experimental values of the masses and P_i are computed with the current value of u as the scan progresses. Figure 2b shows that above 23 MeV the 35 MeV mass unit has the best R^2 of all other local maxima.

The nearby relative maxima correspond to different assignments of the P_i and none of them reproduces the η'/η ratio precisely. The same discrimination is visualized in the chart of the sum of the squares of the residuals (SSR) of the fit as a function of u in figure 2c, where the u value of the minimum near 35 MeV corresponds to the result of the LS fit .

As a curiosity, note that for u = 35 MeV the Gell-Mann-Okubo (GMO) mass formula for the pseudoscalar mesons is an identity in P :

$$(m(\eta))^2 = (4*(m(K))^2-(m(\pi))^2)/3 \;\text{-->}\; 256 = (784-16)/3$$

while for the nearby relative R^2 maxima this is not the case.

3. Mass unit and particle type

The 35 MeV multiplicity hypothesis is generic, and applies to mesons (even multiples of u) as well as baryons and unstable leptons (odd multiples). It is approximate, with a spread of the values of m/P reflecting mass residuals that are large compared to the measurement errors (table 1a). Among the particles with mass below 1 GeV, the η and η' are the only two states with the same quark composition and quantum numbers, and as already noticed in section 2 their m/P coincide to 5 significant digits, *suggesting that the m/P ratio might be a function of the particle type*. In what follows this conjecture will be probed across the complete meson spectrum, by performing a separate multiplicity analysis for each meson family. The baryons will be handled in a separate publication, and the lepton analysis will be appended here at the end.

3.1 Meson sample

The PDG (from now on short for "Review of Particle Physics, by the Particle Data Group" [9]) features a "Michelin star" rating for baryons, from 4 down to 1 star. In the present analysis this rating is extended to the mesons, by assigning 4 stars to the states that are promoted to the summary table and the computer file, 3 stars to those which do not make it, and 2 stars to the light unflavored mesons observed by only one group and needing confirmation, listed in the "further states" section at page 539. The new D_s mesons discovered in 2003 are added to the sample and rated 4 stars.

The combined use of MeV and GeV as the unit of mass may be confusing. The PDG tags most of the mesons with the rounded value of their mass in MeV, e.g. $\eta(1760)$, in the computer file they quote the mass and its error in GeV, while in the printed listing they use mostly MeV apart from a few states where the mass and its error are in GeV, such as the B_c. In what follows masses and errors will be expressed in MeV, short for MeV/c^2.

The values of the multiplicity of the various mesons are computed by dividing the mass by the estimated mass unit, $P_i=m_i/u_i$, therefore a large error errm on m may result in a wrong assignment of P. If the errors on the masses quoted by the PDG were statistical only and gaussian, a cut on errm could be set precisely to correspond to a given value of the confidence level ($1-\alpha$), say 0.95 or 0.99. This not being the case, the cut on errm for a state to be considered in the present analysis is set arbitrarily to 30 MeV. This value will be increased in a few cases where there are not enough states in a given sample, while states with wrong assignments of P that exhibit an abnormally large residual will hopefully be rejected by Chauvenet's criterion (appendix A6). In the tables and related text the states will be identified in a plain format similar to the one used in the PDG computer file, e.g. omega(3)(2250) for $\omega_3(2250)$.

3.2 Example of analysis procedure: the pions

The procedure will be illustrated with the pions, and the results summarized at page 7. Several pages in the same format will follow, showing the results for each meson family. For each family the goals are:

- assess the quantization of the masses and compute u and its error;
- study a possible spin dependence;
- evaluate the p-value of the multiplicity hypothesis: $m_i = P_i * u$;
- try to understand the origin of outliers if any.

Sample selection

The PDG lists 11 pion states, tabulated here at page 7. The '*' column shows the stars rating for each state (4, 3 or 2), while the 'x' column tags states that are excluded from the analysis with the codes:

1 = high errm; **2** = ambiguous PDG record; **3** = Chauvenet;

The pi(+) and pi(0) are averaged in a fictitious state pi(avg), while the pi(1300), with errm=100 MeV, is not used in the analysis (x=1).

Assignment of P , fit of u and goodness-of-fit

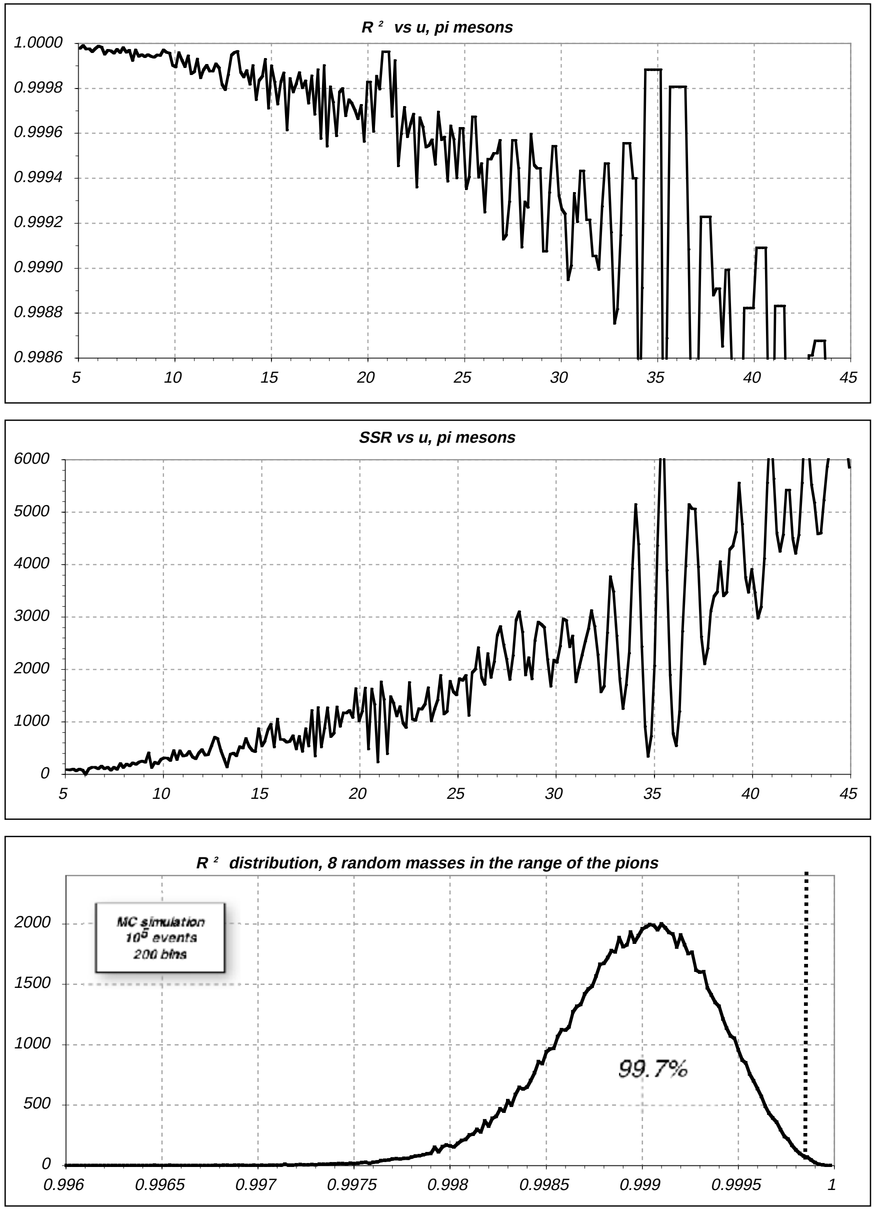
The multiplicity is assessed with a u-scan, by varying u, computing the corresponding P multiplicities from the masses and u, and plotting R^2 and the sum of the squares of the residuals (SSR) as a function of u with the procedure described in section 2. The values of P_i corresponding to the maximum of R^2 in the vicinity of 35 MeV, listed in table 4d, are used to perform a least squares fit m=P*u, with the result: u = 34.78 ±0.076 and R^2 = 0.9997. The p-value of the multiplicity hypothesis, computed by Montecarlo simulation as described in A.4 equals 0.989.

Spin dependence

The pion sample comprises two states with J=0, two with J=1, four with J=2 and one with J=4, and u can be computed separately for the J=0, 1 and 2 subsamples, to check for a possible spin dependence of the mass unit. Here are the results:

J = 0, u = 34.63 ±0.024	Z(0,1) = 0.61
J = 1, u = 34.57 ±0.14	Z(1,2) = 1.66
J = 2, u = 34.75 ±0.064	Z(0,2) = 3.16

to be compared with a threshold of 2.58 (Appendix A.5). Only the (0,2) comparison is indicative of a possible spin dependence, but the value of u for J=0 is influenced by the relatively lower value of m/P of the pi(avg) state. In other families the dependence will be demonstrated with values of Z substantially above threshold. No spin dependence is assumed for the pions.



3a
3b
3c

Fig. 3. u-scan for pions and p-value: 3a, R^2 versus u; 3b, SSR versus u; 3c Montecarlo distribution of R^2 and evaluation of the p-value of the multiplicity hypothesis.

Chauvenet's rejection

The pi(4)(2250) has a residual of 24.2 MeV, more than twice the standard deviation of the residuals of 11.4. Without a firmly established spin dependence of u it would be inconsistent to attribute this large residual of the only J=4 pion state to spin effects. The ratio residual/s.d. amounts to 2.12, and comparing it with a critical deviation of 1.91 for a sample of count=9 it is possible to reject this point by Chauvenet's criterion (Appendix A.6).

The R^2 and SSR distributions are re-evaluated with a new u-scan on a sample of 8 mesons (figures 3a and 3b) identifying the same values for the P_i as the previous scan. A LS fit is performed again on the reduced sample, with the result: u = 34.69 ±0.051, with R^2 = 0.99988. The corresponding p-value of the hypothesis computed by Montecarlo simulation equals 0.997. The residual of the pi(4) against the new value of u is 29.5 MeV.

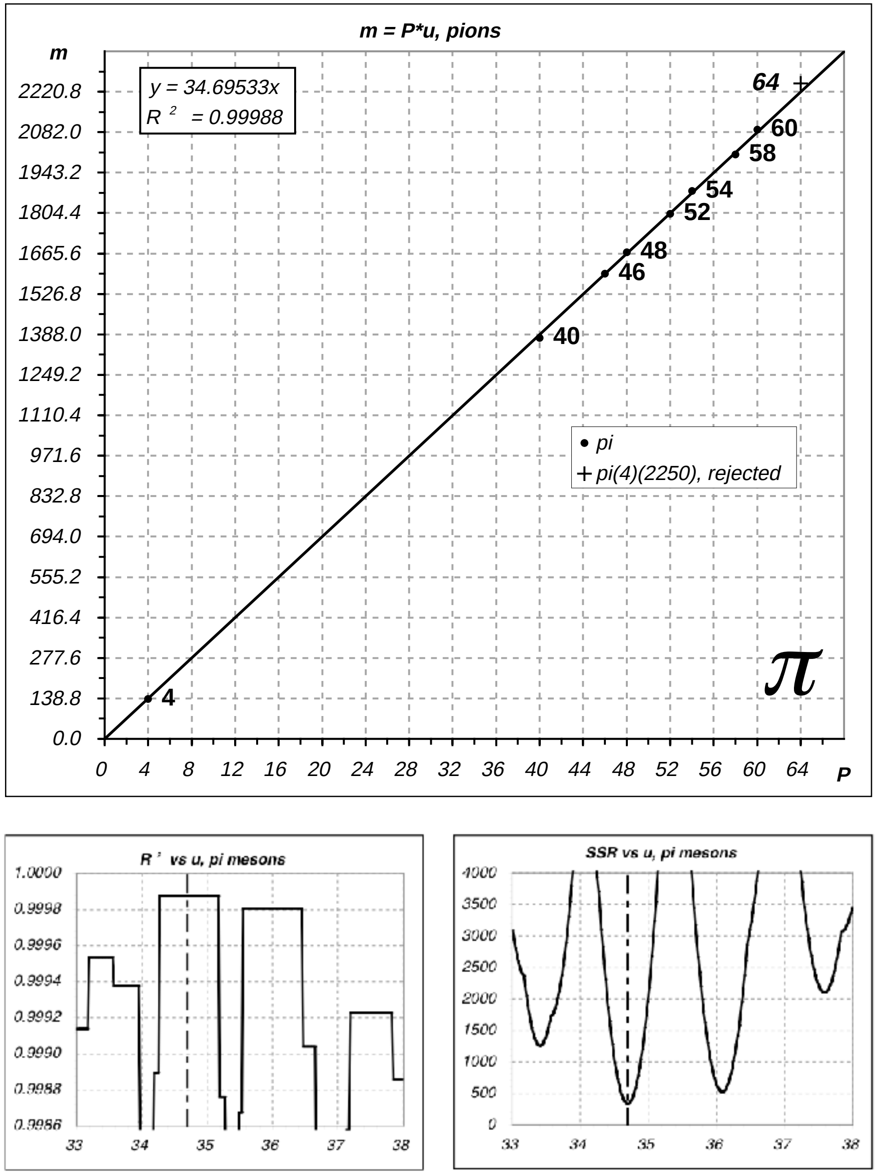
The pi(4)(2250) has a star count of 2 and has been seen in a partial wave analysis of proton-antiproton annihilation. In the same analysis further states were identified, including the pi(2)(2005) that is also part of this pion sample. With the value of the residual not far from the maximum, it is possible that the pi(4) might be an overlap of two nearby states at P=64 and P=66.

Weighted fit

A weighted LS fit using the PDG errors on the masses is performed iteratively, starting with the full sample and removing states one by one starting at low P (appendix A.3). The fit result and the corresponding $\Sigma_i(\text{residual}_i/\text{errm}_i)^2$ stabilize after removing the state at P=4 (plot not shown) at the value uw = 34.65 ±0.051. The pions at P=4 are measured with better precision then the other states, and the mass used in the fit is averaged. If there were a deviation from linearity at low P the result would be biased, and the fitting procedure described above avoids this problem. For the various meson families the value of uw will be reported in the summaries only if significantly different from u, and qualified by a possible omission of low masses. The dependence of u from P within a family, the variation of u with the spin (if any), and the mass differences within I-multiplets will be addressed in a separate publication.

Pions summary

Page 7 presents the results of the analysis for the pion family. It features the m vs P plot, with the value of u from the LS fit as well as the R^2, the u-scan R^2 and SSR plots restricted to the 33-38 MeV interval, a table with all the states and a summary. One such page will follow for each meson family. (In the upper left box of the m vs P plot only 2 digits of the slope are significant after the decimal point, but more are displayed due to an excel formatting limitation. More digits are needed for the R^2).



4a

4b

4c

Fig. 4. Mass multiplicity, pi mesons: 4a, m vs P and line fit, ad-hoc mass scale; 4b, R^2 vs u; 4c, SSR vs u.

4. Mass unit analysis by meson family

4.1 The pions

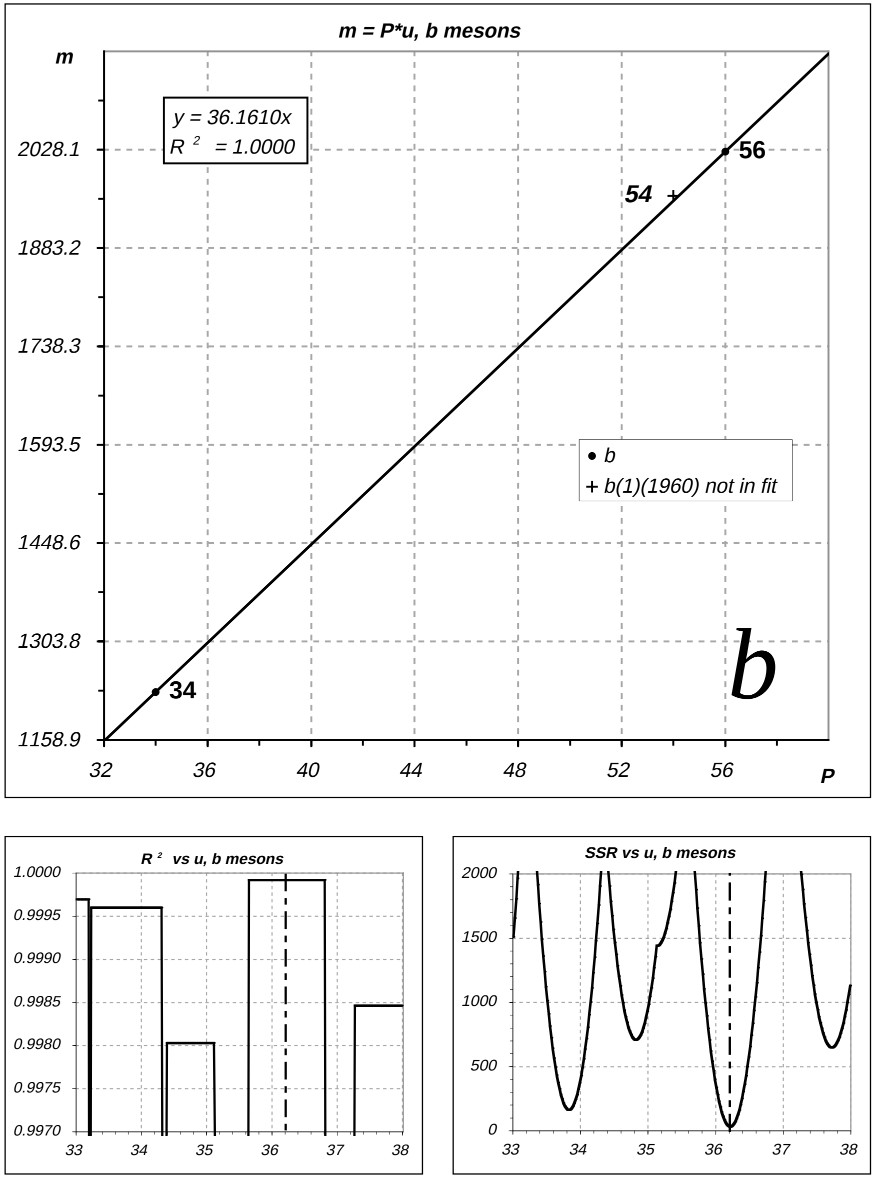
8 pions, including two “non q-qbar” J=1 states define a sharp multiplicity alignment with u = 34.69 and a p-value of 0.997. Fitting separately states with J = 0, 1 and 2 does not show any statistically significant spin dependence. The result of the fit is stable if the pi(avg) state is removed and also if the sample is restricted to 4-star states only.

The pi(1300) with an error of 100 MeV is excluded from the sample. The pi(4)(2250) has a large residual that it would be inconsistent to attribute to spin effects, and it can be rejected by Chauvenet's criterion. It is the only J=4 pion known, and the result of the observation by only one group. A residual of 29.5 is very close to the maximum, and indicates that the pi(4) might be an overlap of two 4^- states at P=64 and 66. This cannot be excluded by the experimental data [10].

Removing the pi(4) for the sample decreases the value of u by 0.13 MeV, reduces the error on u by 30% and improves marginally the already very good p-value of the hypothesis.

meson type = pi										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
pi(avg 0/+)	4	0,+	0		4	137.3	6.0E-04	34.318	-1.8	1.33%
pi(1300)	4	0,+	0	1	38	1300.0	100.0	34.211	-18.4	1.42%
pi(1)(1400)	3	0,+	1		40	1376.0	17.0	34.400	-14.9	1.09%
pi(1)(1600)	3	0,+	1		46	1596.0	20.0	34.696	-3.6	0.22%
pi(2)(1670)	4	0,+	2		48	1670.0	20.0	34.792	0.9	0.05%
pi(1800)	4	0,+	0		52	1801.0	13.0	34.635	-7.2	0.40%
pi(2)(1880)	2	0,+	2		54	1880.0	20.0	34.815	2.2	0.12%
pi(2)(2005)	2	0,+	2		58	2005.0	15.0	34.569	-11.9	0.59%
pi(2)(2100)	3	0,+	2		60	2090.0	29.0	34.833	3.6	0.17%
pi(4)(2250)	2	0,+	4	3	64	2250.0	15.0	35.156	24.5	1.09%

summary pi mesons	
u	34.69 ± 0.051
p-value	0.997
spin dependence	no
omitted	3 = 1 averaged + 1 large errm + 1 Chauvenet



5a

5b

5c

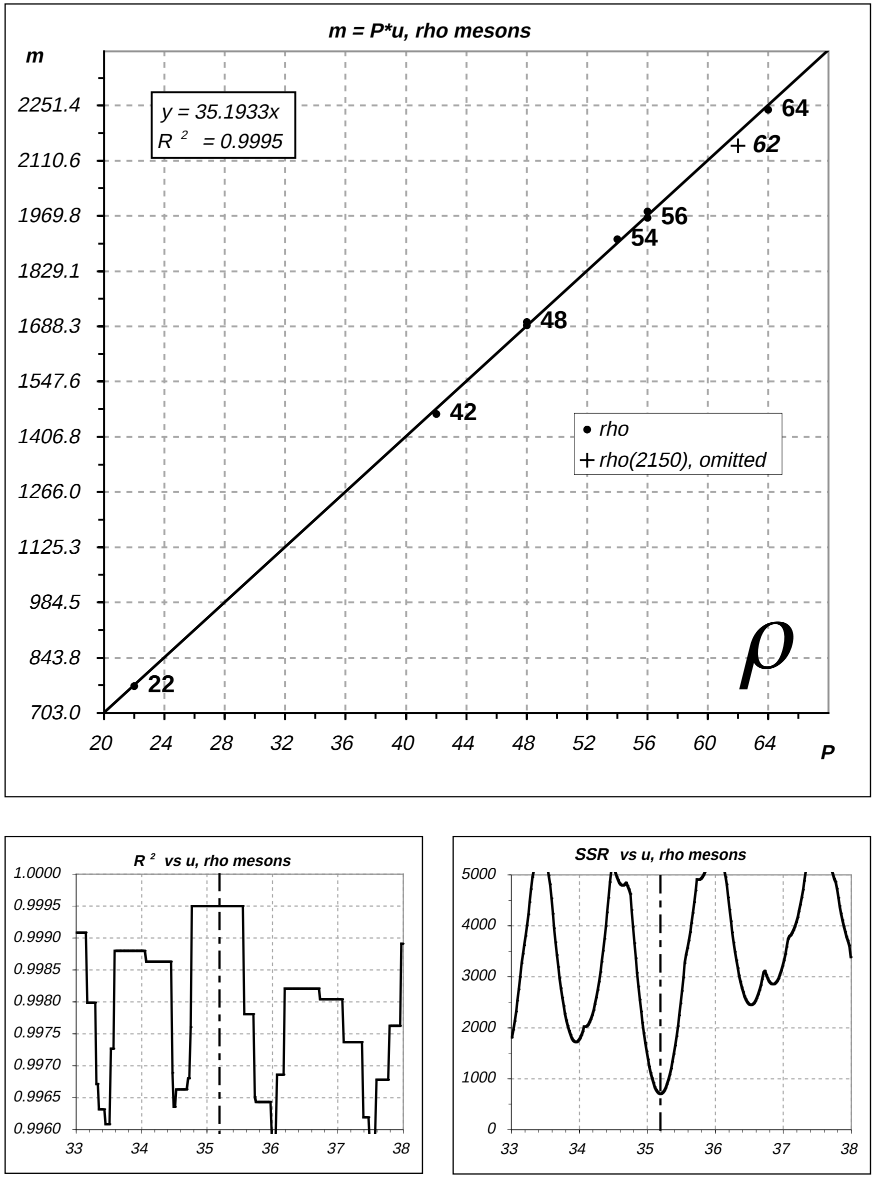
Fig. 5. Mass multiplicity, b mesons: 5a, m vs P and line fit, ad-hoc mass scale; 5b, R^2 vs u; 5c, SSR vs u;

4.2 The b mesons

Apart from the b(1)(1235), two more b mesons can be found in the “further states” section of the PDG. The b(1)(1960) is listed with an error on the mass of 40 MeV, while the b(3)(2025) has a small error and is interesting in order to test the spin dependence. A u-scan with the three states sets u at 36.22 MeV, with a p-value of 0.990, and the J=3 state is well aligned with the other two, consistent with no J dependence of u. To get a more precise value of u, the b(1)(1960) is removed from the sample because of its high errm, and the final value of u is computed with a LS fit using the two remaining states. The effect on the fit is not significant.

meson type = b										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
b(1)(1235)	4	0,+	1		34	1229.5	3.2	36.162	-1.8	-0.15%
b(3)(2025)	2	0,+	3		56	2025.0	15.0	36.161	-3.1	-0.15%
b(1)(1960)	2	0,+	1	1	54	1960.0	40.0	36.296	4.4	0.22%

summary b mesons	
u	36.16 ± 0.047
p-value	0.990
spin dependence	no
omitted	1 large errm



6a 6b 6c

Fig. 6. Mass multiplicity, rho mesons: 6a, m vs P and line fit, ad-hoc mass scale; 6b, R^2 vs u; 6c, SSR vs u;

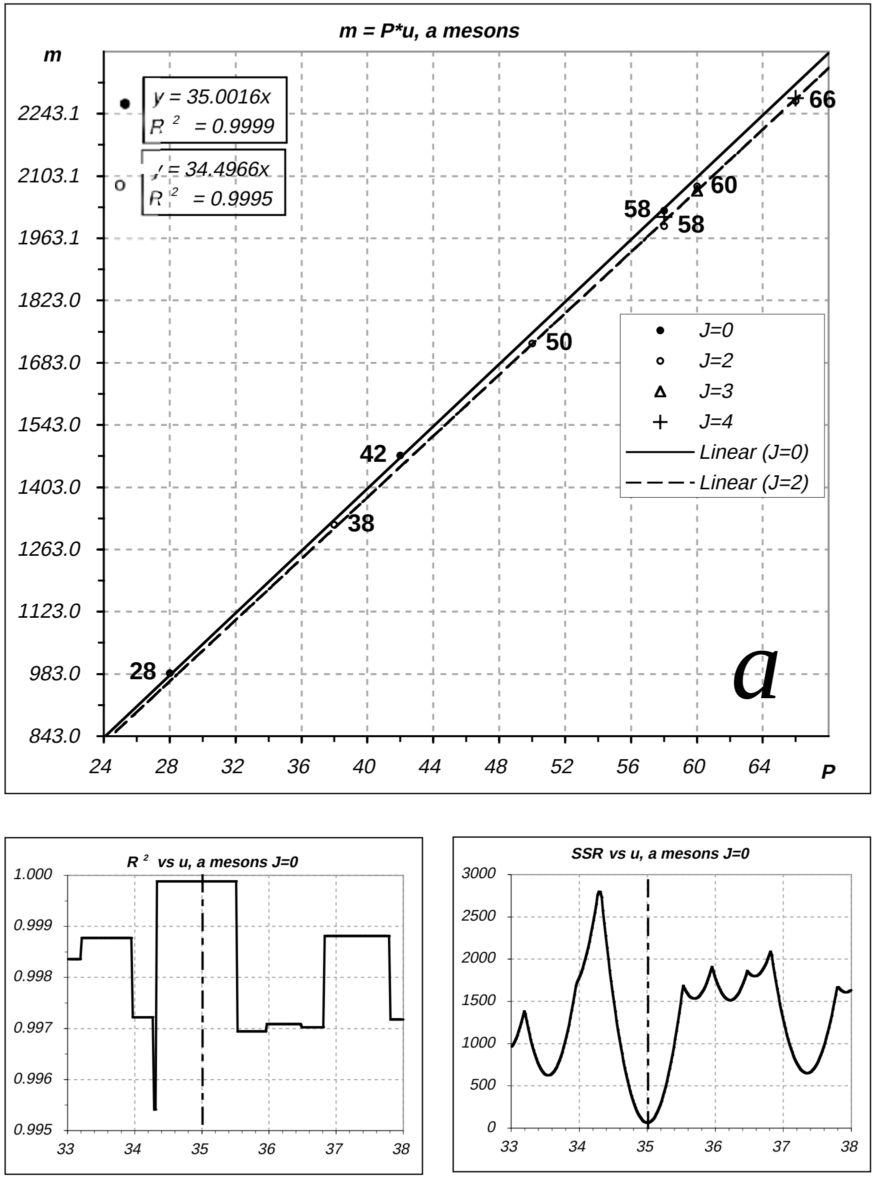
4.3 The rho mesons

The rho family shows a good multiplicity alignment, no spin dependence and 1 state with a large residual. The rho(3)(2250) and rho(5)(2350) masses have large errors and are not considered. Separate fits of states with J=1 and 3 do not show any statistically significant spin dependence. The rho(2150) has a large residual and it is rejected by Chauvenet's criterion. A residual of about 33 MeV makes it a suspect for being an overlap of two states with the same quantum numbers and dP=2. From the PDG listing it appears that this state, formerly called the T(1)(2190), might be an averaging overlap of the "old" state at 2190 MeV and a lighter state at 2115. If this were the case, both states would fit on the line with P=62 and 60 respectively, with their overlap showing a residual close to u.

The value of u does not vary significantly be removing the rho(770) from the sample, nor by considering only 4 stars states. A weighted fit gives u = 35.31 ±0.044 omitting the rho(770).

meson type = rho										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
rho(770)	4	0,+	1		22	771.1	0.9	35.050	-3.2	-0.42%
rho(1450)	4	0,+	1		42	1465.0	25.0	34.881	-13.3	-0.91%
rho(3)(1690)	4	0,+	3		48	1691.0	5.0	35.229	1.6	0.09%
rho(1700)	4	0,+	1		48	1700.0	20.0	35.417	10.6	0.62%
rho(1900)	3	0,+	1		54	1911.0	5.0	35.389	10.4	0.54%
rho(1965)	2	0,+	1		56	1965.0	30.0	35.089	-6.0	-0.31%
rho(3)(1990)	3	0,+	3		56	1981.0	14.0	35.375	10.0	0.50%
rho(2150)	3	0,+	1	3	62	2149.0	16.0	34.677	-33.2	-1.54%
rho(4)(2240)	2	0,+	4		64	2240.0	25.0	35.000	-12.6	-0.56%
rho(3)(2250)	3	0,+	3	1	64	2250.0	200.0	35.156	-2.6	-0.12%
rho(5)(2350)	3	0,+	5	1	66	2330.0	35.0	35.303	7.0	0.30%

summary rho mesons	
u	35.19 ± 0.071 -- weighted: 35.31 ±0.044
p-value	0.973
spin dependence	no
omitted	3 = 2 large errm + 1 Chauvenet



7a

7b

7c

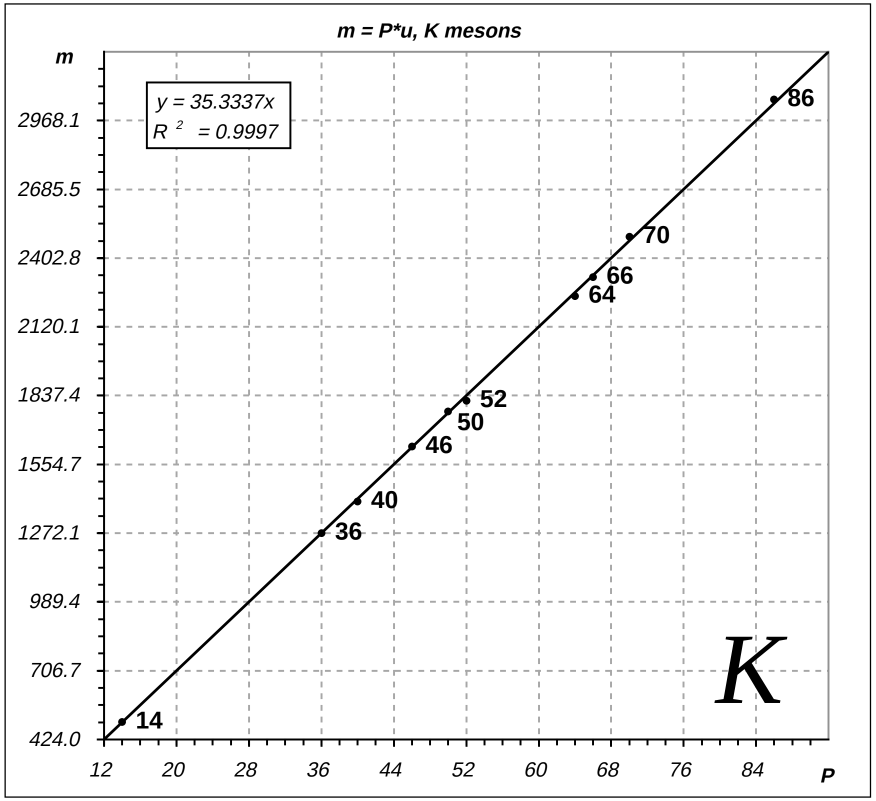
Fig. 7. Mass multiplicity, a mesons: 7a, m vs P and line fit, ad-hoc mass scale; 7b, R^2 vs u; 7c, SSR vs u;

4.4 The a mesons

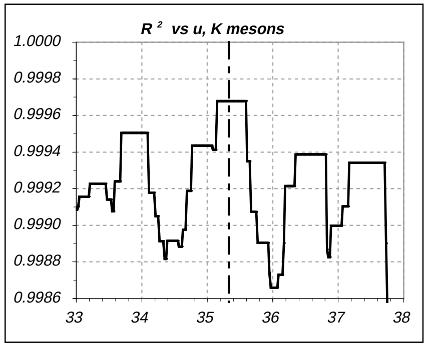
All a mesons are considered, including non q-qbar states. The a(1)(1260) and a(6)(2450) masses have large errors and are omitted. The u-scan with the sample of the remaining 11 states features u = 34.61 and an R^2 of 0.9991 for a p-value of 0.995. Separate fits of states with J=0 and 2 show a significant spin dependence (Z=9.6). The LS fit of the J=0 sample gives u = 35.00 ±0.073, with a p-value of 0.941. The same sample fitted with weights gives u = 35.16, due to the influence of the a(0)(980) measured with a better precision than the other two J=0 states.

meson type = a										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
a(0)(980)	4	0,+	0		28	984.7	1.2	35.168	4.7	0.47%
a(1)(1260)	4	0,+	1	1	36	1230.0	40.0	34.167	-15.0	-1.22%
a(2)(1320)	4	0,+	2		38	1318.0	0.6	34.684	7.1	0.54%
a(0)(1450)	4	0,+	0		42	1474.0	19.0	35.095	3.9	0.27%
a(2)(1700)	3	0,+	2		50	1726.0	26.0	34.520	1.2	0.07%
a(2)(1990)	2	0,+	2		58	1990.0	22.0	34.310	-10.8	-0.54%
a(4)(2040)	4	0,+	4		58	2011.0	13.0	34.672	4.2	0.21%
a(0)(2020)	2	0,+	0		58	2025.0	30.0	34.914	-5.1	-0.25%
a(3)(2070)	2	0,+	3		60	2070.0	20.0	34.500	-5.1	-0.25%
a(2)(2080)	2	0,+	2		60	2080.0	20.0	34.667	10.2	0.49%
a(2)(2270)	2	0,+	2		66	2272.0	20.0	34.424	-4.8	-0.21%
a(4)(2280)	2	0,+	4		66	2280.0	15.0	34.545	-3.7	-0.16%
a(6)(2450)	3	0,+	6	1	70	2450.0	130.0	35.000	27.9	1.14%

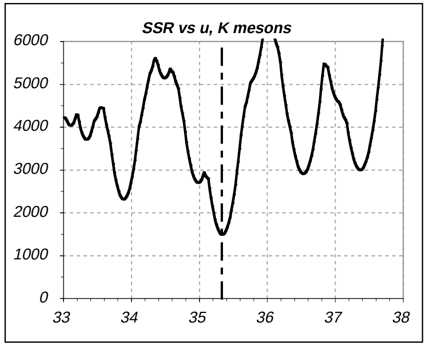
summary a mesons	
u, J=0	35.00 ± 0.073 -- weighted 35.16 ± 0.015
p-value	> 0.995 for all states, = 0.941 for a(0)
spin dependence	yes, Z=9.64
omitted	2 large errm



8a



8b



8c

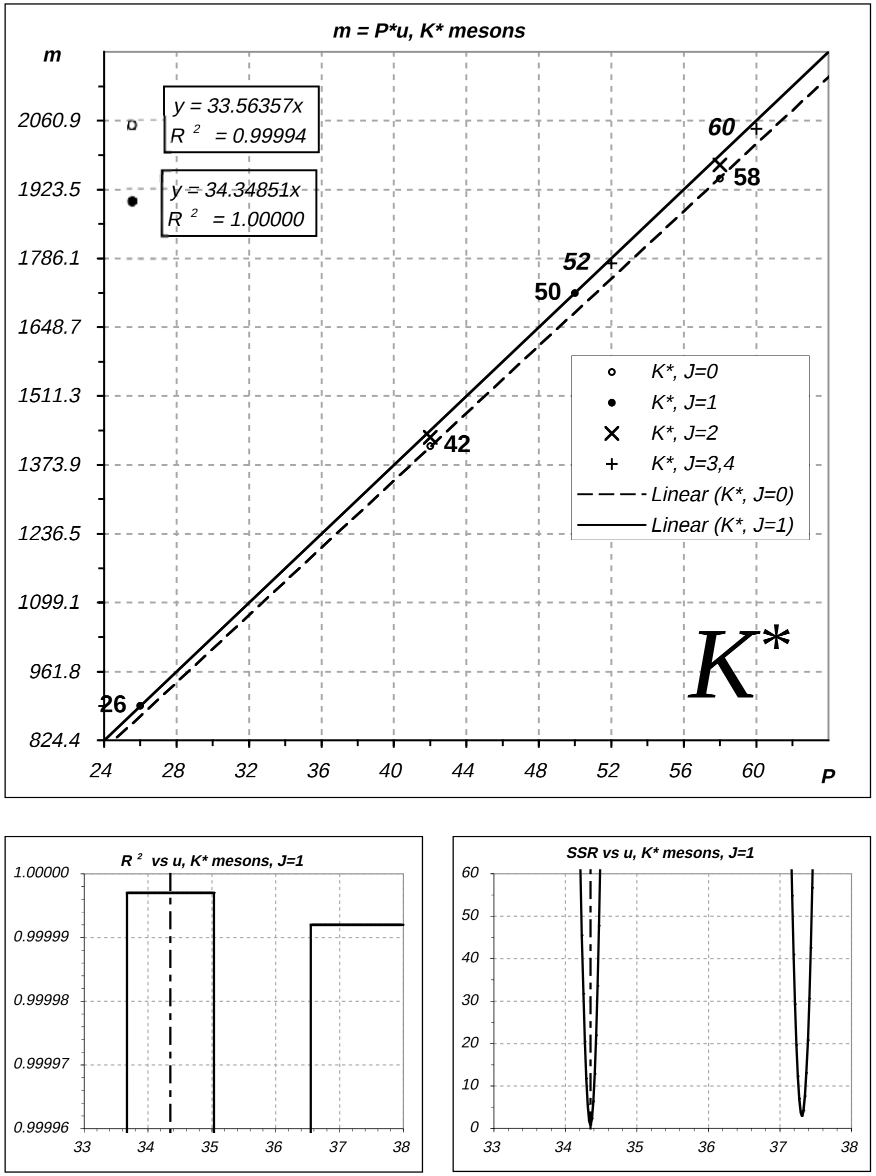
Fig. 8. Mass multiplicity, K mesons: 8a, m vs P and line fit, ad-hoc mass scale; 8b, R^2 vs u; 8c, SSR vs u;

4.5 The K mesons

Of the 11 K states, 10 can be retained after averaging the K^+ and K^0 in the fictitious state K(avg). The LS fit yields u = 35.33 ±0.075, with an R^2 corresponding to a p-value of 0.943. Separate fits for J=0, 1 and 2 do not reveal a significant spin dependence, and u does not change removing the K(avg) from the sample. The residuals are comparable with the mass errors, while for several other families they are smaller. This is reflected in the comparatively larger error on u and in a good but not outstanding p-value. The weighted fit omitting the K(avg) is compatible with the non-weighted result. By reducing the sample to the 4-star states, u equals 35.19 ±0.117.

meson type = K										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u=m/P**	**dm**	**dm/m**
K(avg)	4	0,+	0		14	495.7	4.0E-02	35.405	3.0	0.20%
K(1)(1270)	4	0,+	1		36	1273.0	7.0	35.361	6.2	-0.01%
K(1)(1400)	4	0,+	1		40	1402.0	7.0	35.050	-5.6	-0.81%
K(2)(1770)	4	0,+	2		50	1773.0	8.0	35.460	13.5	0.35%
K(2)(1820)	4	0,+	2		52	1816.0	13.0	34.923	-13.8	-1.18%
K(1630)	3	0,+	0		46	1629.0	7.0	35.413	10.3	0.10%
K(2)(2250)	3	0,+	2		64	2247.0	17.0	35.109	-5.1	-0.64%
K(3)(2320)	3	0,+	3		66	2324.0	24.0	35.212	1.5	-0.35%
K(4)(2500)	3	0,+	4		70	2490.0	20.0	35.571	26.8	0.66%
K(3100)	3	0,+	?		86	3054.0	11.0	35.512	27.7	0.56%

summary K mesons	
u	35.33 ± 0.075
p-value	0.943
spin dependence	no
omitted	1, averaging



9a 9b 9c

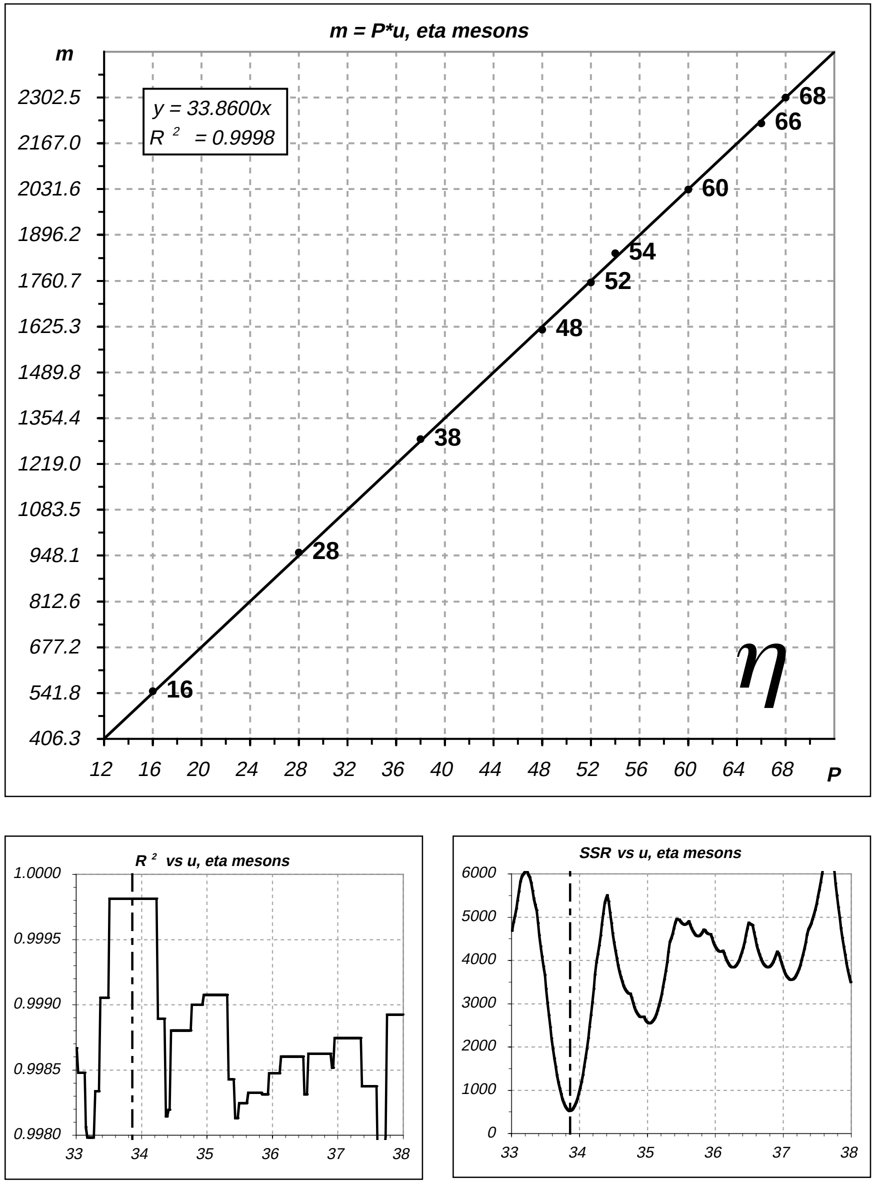
Fig. 9. Mass multiplicity, K* mesons: 9a, m vs P and line fit, ad-hoc mass scale; 9b, R^2 vs u; 9c, SSR vs u;

4.6 The K* mesons

The PDG lists 12 K* mesons, and the sample is reduced to 8 by averaging two isospin multiplets and omitting 2 states with errm> 30. The u-scan and LS fit give u = 33.98 but the R^2 is not very good and the corresponding p-value equals 0.883 only. Fitting separately the samples with J=0, 1 (and also 2 after rescuing an omitted state with errm=33) gives a clear indication of spin dependence, with Z(0,1) = 25.6 and Z(1,2) = 27.9. In the J=1 sample, the K*(1410) has a very large residual and can be omitted by Chauvenet's criterion (it is intriguing that the K*(1410) is very close in mass to the K*(0)(1430), only 2 MeV).The LS fit for J=1 gives u = 34.35 ±0.016

meson type = K*										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
K*(892)avg	4	0.+	1		26	893.9	0.3	34.380	0.8	0.09%
K*(0)(1430)	4	0,+	0		42	1412.0	6.0	33.619	2.3	0.17%
K*(1410)	4	0,+	1	3	42	1414.0	15.0	33.667	-28.6	-2.03%
K*(2)(1430)avg	4	0,+	2		42	1429.0	1.5	34.024	0.2	0.01%
K*(1680)	4	0,+	1		50	1717.0	27.0	34.340	-0.4	-0.02%
K*(3)(1780)	4	0,+	3		52	1776.0	7.0	34.154		
K*(0)(1950)	3	0,+	0		58	1945.0	30.0	33.534	-1.7	-0.09%
K*(2)(1980)	3	0,+	2		58	1973.0	33.0	34.017	-0.1	-0.01%
K*(4)(2045)	4	0,+	4		60	2045.0	9.0	34.083		
K*(5)(2380)	3	0,+	5	1	70	2382.0	33.0	34.029		

summary K* mesons	
u, J=1	34.35 ± 0.016
p-value	> 0.882 (value for all states together)
spin dependence	yes, Z=25.59 and 27.52
omitted	4 = 2 averaged +1 large errm + 1 Chauvenet



10a

10b

10c

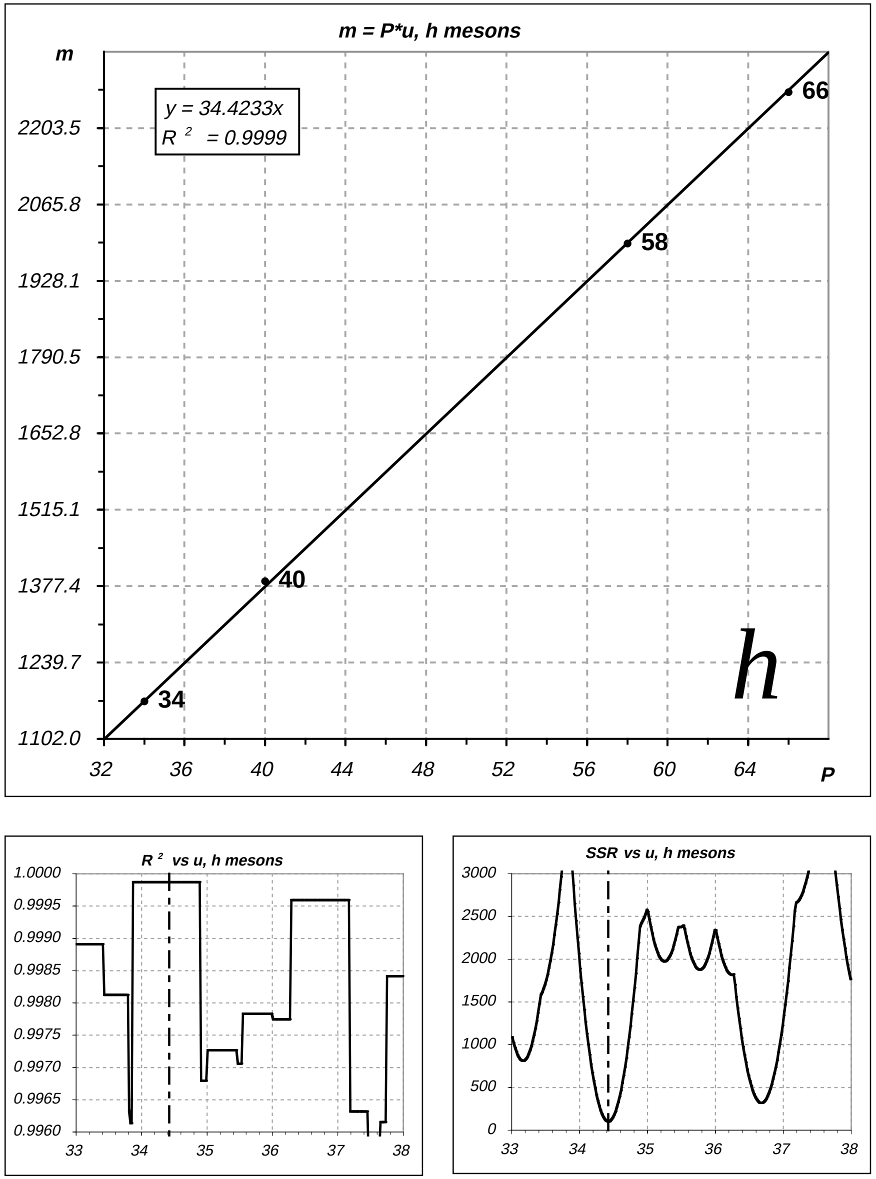
Fig. 10. Mass multiplicity, eta mesons: 10a, m vs P and line fit, ad-hoc mass scale; 10b, R^2 vs u; 10c, SSR vs u;

4.7 The eta mesons

The multiplicity p-value of the eta mesons is an outstanding 0.999, and the m vs P alignment is spectacular. From a sample of 13 states, 4 are removed because of large errors. The u-scan identifies a sharp alignment at u = 33.86, with an R^2 corresponding to a p-value of 0.999. Interestingly the result of the fit is significantly lower by 0.35 MeV in comparison with the value of m/P of the low-mass eta and eta'(958) at P=16 and 28, while the values for these two states are the same with higher precision. Separate fits of J=0 and J=2 states show no spin dependence. A weighted fit omitting the two etas at low mass finds the same value for u as the non-weighted LS fit.

meson type = eta										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
eta	4	0	0		16	547.3	0.1	34.206	5.5	1.01%
eta'(958)	4	0	0		28	957.8	0.1	34.206	9.7	1.01%
eta(1295)	4	0	0		38	1293.0	5.0	34.026	6.3	0.49%
eta(1440)	4	0	0	1	42	1435.0	35.0	34.167	12.9	0.90%
eta(2)(1645)	3	0	2		48	1617.0	5.0	33.688	-8.3	-0.51%
eta(1760)	3	0	0		52	1756.0	11.0	33.769	-4.7	-0.27%
eta(2)(1870)	3	0	2		54	1842.0	8.0	34.111	13.6	0.74%
eta(2)(2030)	2	0	2		60	2030.0	20.0	33.833	-1.6	-0.08%
eta(2190)	2	0	0	1	64	2190.0	50.0	34.219	23.0	1.05%
eta(2)(2250)	2	0	2		66	2225.8	13.0	33.723	-9.0	-0.40%
eta(2225)	3	0	0	1	66	2227.0	35.0	33.742	-7.8	-0.35%
eta(2280)	3	0	0		68	2302.5	12.0	33.860	0.0	0.00%
eta(4)(2320)	2	0	4	1	68	2328.0	38.0	34.235	25.5	1.10%

summary eta mesons	
u	33.86 ± 0.053
p-value	0.999
spin dependence	no
omitted	4 large errm



11a

11b

11c

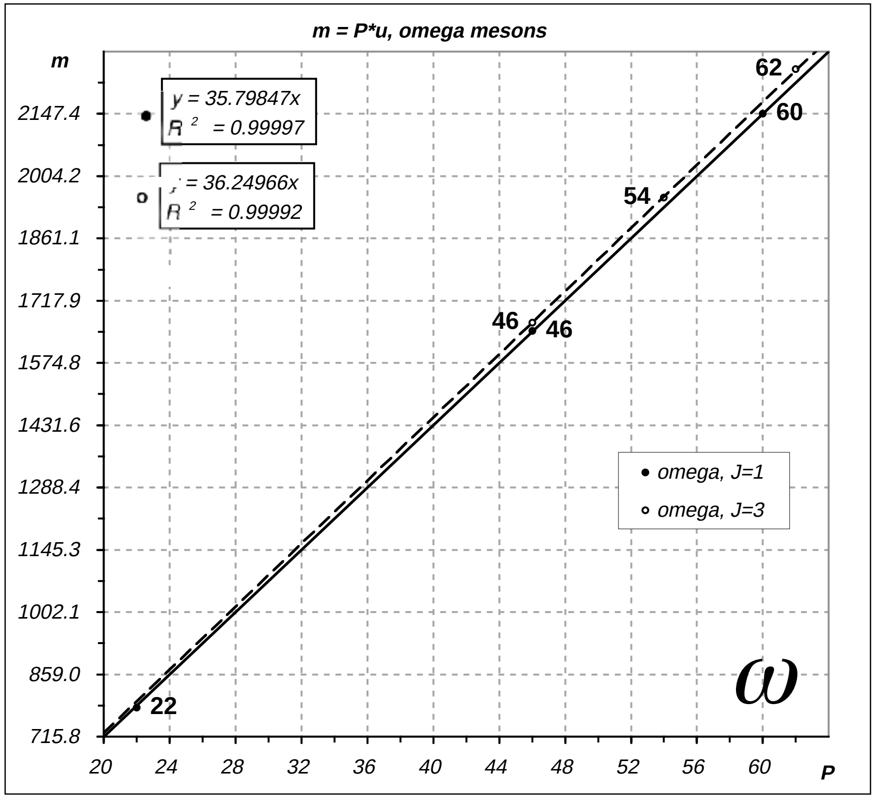
Fig. 11. Mass multiplicity, h mesons: 11a, m vs P and line fit, ad-hoc mass scale; 11b, R^2 vs u; 11c, SSR vs u;

4.8 The h mesons

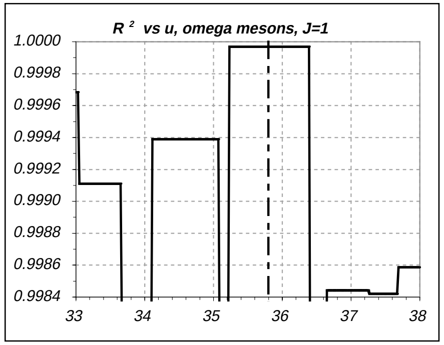
With the inclusion of three 2-star states the h meson sample is populated to a total count of 6, then two mesons with errm > 30 MeV are omitted leaving 4 states all with J=1. The u-scan and fit procedure finds u = 34.42 ±0.056, with an R^2 corresponding to a p-value of 0.975. The multiplicity alignment is very sharp, with residuals that are substantially smaller than the quoted errors on the masses. For this reason it is meaningful to re-include in the sample the discarded h(3) state at P=66 with errm=35 MeV, and deduce that u is likely not spin dependent on the basis of its residual of only 3.07 MeV with respect to the h(1) multiplicity line.

meson type = h										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
h(1)(1170)	4	0	1		34	1170.0	20.0	34.412	-0.4	-0.03%
h(1)(1380)	3	0	1		40	1386.0	19.0	34.650	9.1	0.65%
h(1)(1595)	3	0	1	1	46	1594.0	50.0	34.652	10.5	0.66%
h(1)(1995)	2	0	1		58	1995.0	20.0	34.397	-1.5	-0.08%
h(1)(2265)	2	0	1		66	2268.0	20.0	34.364	-3.9	-0.17%
h(3)(2275)	2	0	3	1	66	2275.0	35.0	34.470	3.1	0.13%

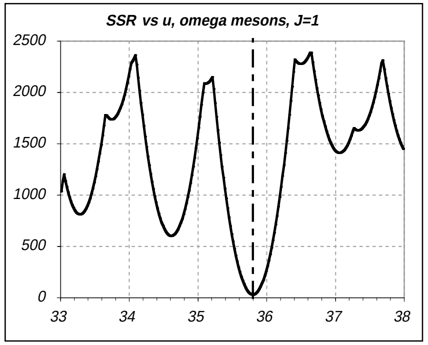
summary h mesons	
u	34.42 ± 0.056
p-value	0.975
spin dependence	no, judging from the residual of the omitted h(3)(2275)
omitted	2 large errm



12a



12b



12c

Fig. 12. Mass multiplicity, omega mesons: 12a, m vs P and line fit, ad-hoc mass scale; 12b, R^2 vs u; 12c, SSR vs u;

4.9 The omega mesons

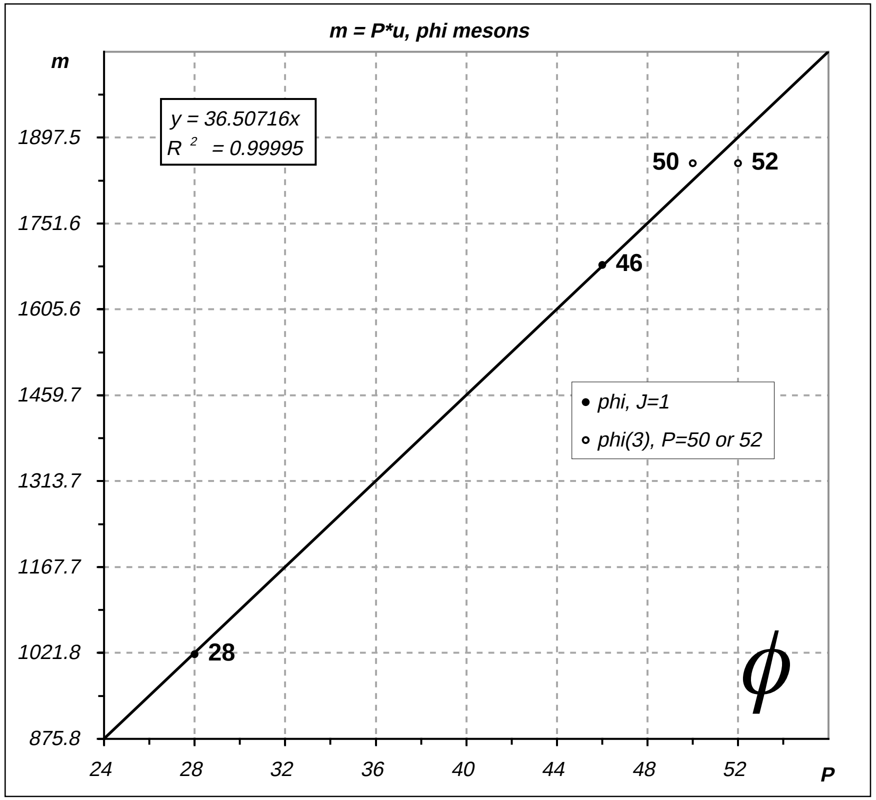
The analysis of the omega mesons shows a clear indication of spin-dependent multiplicity. Out of a sample of 7 states, 1 has errm > 30 MeV and is omitted. Fitting separately the three J=1 states and the three with J=3, a spin dependence is manifest, with Z=13.9. The result of the LS fit for J=1 is u = 35.80 ±0.049, and the weighted fit yields the same result omitting the low-errm P=22 omega(782). The J=3 weighted and not weighted fits converge to the same value of u.

meson type = omega										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u=m/P**	**dm**	**dm/m**
omega(782)	4	0	1		22	782.6	0.1	35.571	-5.0	-0.64%
omega(1420)	4	0	1	1	40	1419.0	31.0	35.475	-12.9	-0.91%
omega(1650)	4	0	1		46	1649.0	24.0	35.848	2.3	0.14%
omega(3)(1670)	4	0	3		46	1667.0	4.0	36.239	-0.5	-0.03%
omega(3)(1995)	2	0	3		54	1955.0	30.0	36.204	-2.5	-0.13%
omega(2145)	2	0	1		60	2148.0	15.0	35.800	0.1	0.00%
omega(3)(2250)	2	0	3		62	2250.0	20.0	36.290	2.5	0.11%

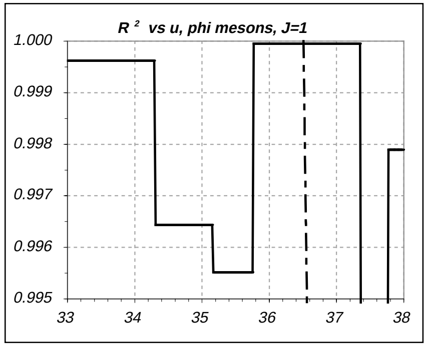
summary omega mesons	
omitted	1 large errm
spin dependence	yes, Z=13.9
u, J=1	35.80 ± 0.049
p-value	> 0.934 (all states), 0.942 for J=1, 0.947 for J=3

4.10 The phi mesons

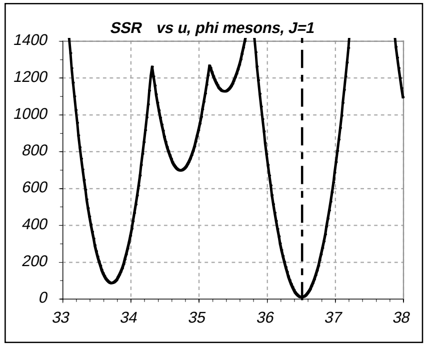
The u-scan with the three phi states identifies the best u = 36.77, unambiguous but with a poor $R^2 = 0.9988$ and a p-value of only 0.732. Restricting the u-scan to the two J=1 states yields u = 36.51 ±0.050 with $R^2 = 0.9999$, while the corresponding p-value is not estimated. A dependence of u from the spin can be assumed, as is the case with the omega family. The weighted fit with the two J=1 states finds u = 36.41, 0.1 MeV below the non-weighted value.



13a



13b



13c

Fig. 13. Mass multiplicity, phi mesons: 13a, m vs P and line fit, ad-hoc mass scale; 13b, R^2 vs u; 13c, SSR vs u;

meson type = phi										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
phi(1020)	4	0	1		28	1019.5	2.0E-02	36.409	-2.3	-0.23%
phi(1680)	4	0	1		46	1680.0	20.0	36.522	1.4	0.08%
phi(3)(1850)	4	0	3		50	1854.0	7.0	37.080	29.4	1.59%

summary phi mesons	
omitted	none
spin dependence	yes
u, J=1	36.51 ± 0.050 MeV -- weighted = 36.41
p-value	> 0.732

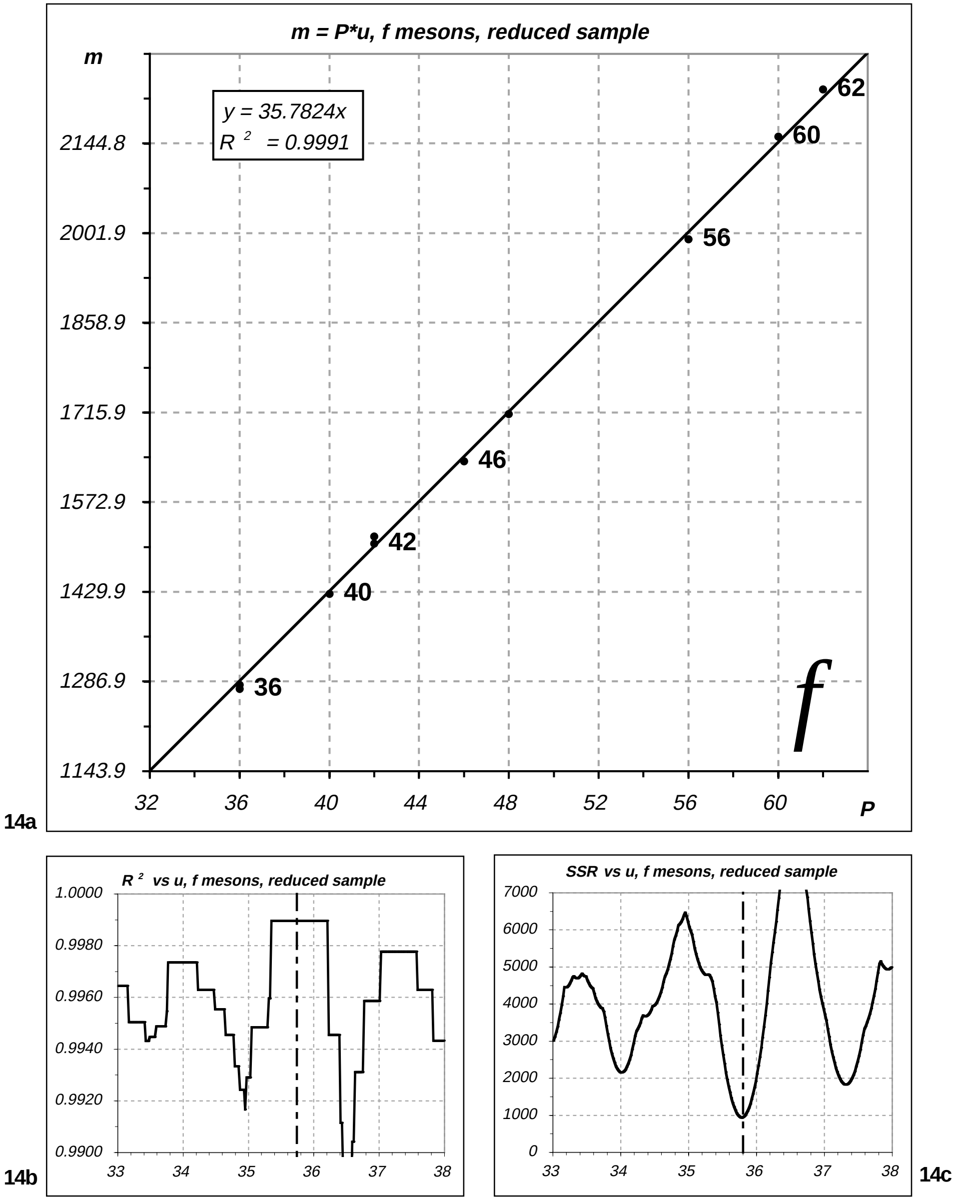


Fig. 14. Mass multiplicity, f mesons: 14a, m vs P and line fit, ad-hoc mass scale; 14b, R^2 vs u; 14c, SSR vs u;

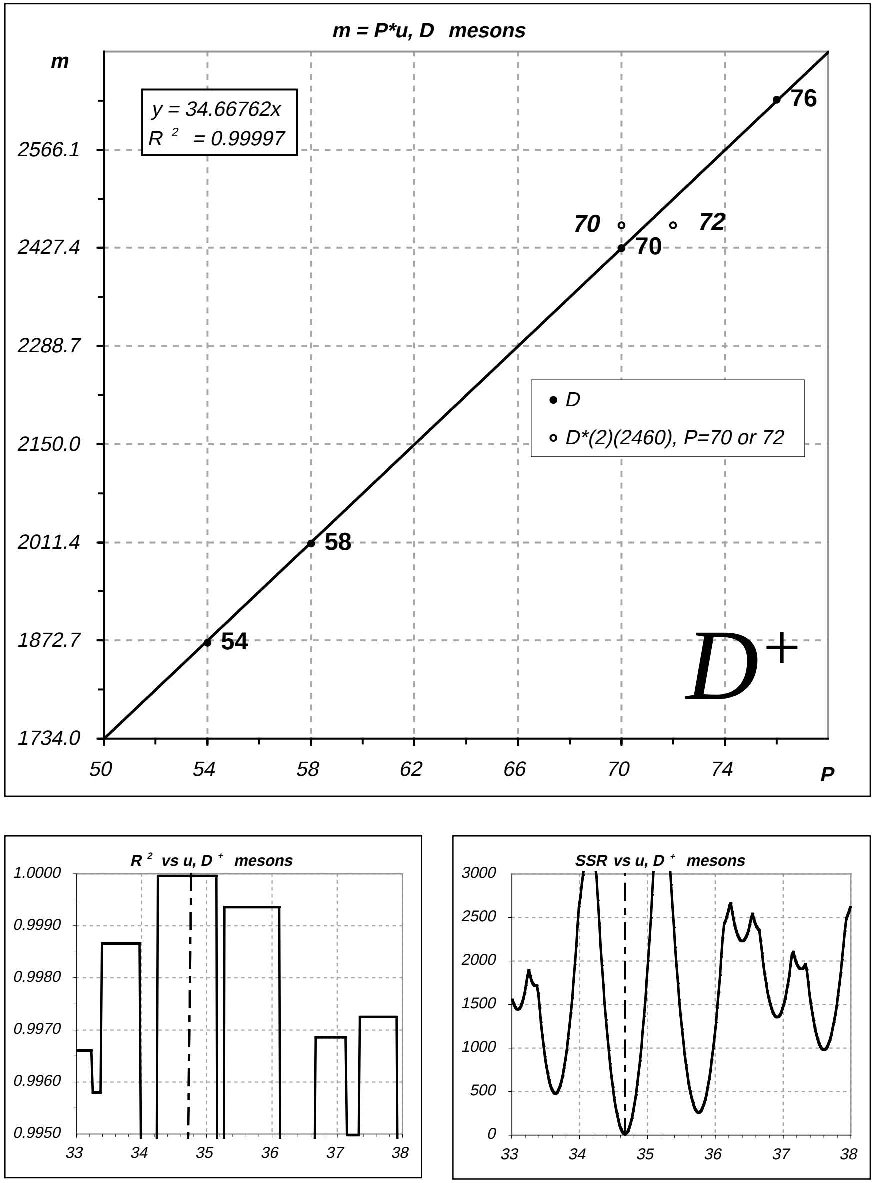
4.11 The f mesons

The analysis of the crowded f family is problematic. This may be due to a combination of measurement difficulties and there being very many states, so that the risk of overlap is high. Of the 33 f states reported by the PDG, 5 have large errm and the masses of 3 more are not well defined. A u-scan with the remaining 25 states (plot not shown) peaks at u=35.63, but with a p-value of only 0.815. A separate fit of J=0,1 and 2 shows a significantly larger value of u for J=1 (with Z=4.70 compared to J=0 and Z=3.85 compared to J=2), but erru is large and the R^2 is not so good.

By removing from the sample all states with mass "estimated" by the PDG rather than averaged or fitted through a statistical procedure, and also all the 2-star states, the sample is reduced to a count of 10. The LS fit of the reduced sample gives u = 35.78 ±0.068 a much better p-value of 0.993 but the fit result is not stable if the low-mass states are removed one by one, while the J dependence is no longer present. Reducing the sample to the five 4-stars states only (plots not shown) yields u=35.67 ±0.070 with a p-value of 0.986. While the multiplicity is well established, a dependence of u from J cannot be excluded, and the value of u should carry also a systematic error of order 0.15 MeV.

meson type = f, reduced sample										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
f(2)(1270)	4	0	2		36	1275.4	1.2	35.428	-12.4	-0.97%
f(1)(1285)	4	0	1		36	1281.9	0.6	35.608	-5.9	-0.46%
f(1)(1420)	4	0	1		40	1426.3	1.1	35.658	-4.6	-0.32%
f(0)(1500)	4	0	0		42	1507.0	5.0	35.881	4.5	0.30%
f(1)(1510)	3	0	1		42	1518.0	5.0	36.143	15.5	1.02%
f(2)(1640)	3	0	2		46	1638.0	6.0	35.609	-7.5	-0.46%
f(0)(1710)	4	0	0		48	1713.0	6.0	35.688	-4.1	-0.24%
f(0)(2020)	3	0	0		56	1992.0	16.0	35.571	-11.3	-0.57%
f(2)(2150)	3	0	2		60	2156.0	11.0	35.933	9.6	0.45%
f(J)(2220)	3	0	?		62	2231.1	3.5	35.985	13.2	0.59%

summary f mesons	
u	35.78 ± 0.068 ± 0.15
p-value	0.998
spin dependence	not excluded
omitted	23 = 5 large errm + 18 dubious



15a 15b 15c

Fig. 15. Mass multiplicity, D+ mesons: 15a, m vs P and line fit, ad-hoc mass scale; 15b, R^2 vs u; 15c, SSR vs u;

4.12 The D and D* mesons

The masses of the D mesons are known with good precision and, out of 5 known charged D and D* mesons, 4 corresponding neutral states have been measured so that a separate analysis for charged and neutral states is possible. The PDG actually lists 2 $D^{\pm}$ states and their neutral companions, and also 3 $D^{*\pm}$ of which 2 with the corresponding neutral. Considering the J^P assignments of the 5 charged states, they are well measured for the D charged and neutral, tentative for the neutral D(1)(2420) and only a guess for the corresponding charged state. As to the D* states, the D*(2)(2460) is definitely a 2^+ and its mass has been confirmed recently by the FOCUS collaboration, while the two other D* states are listed with J^P to be confirmed, so that they could be either D or D*.

In the pseudoscalar K family u does not vary with the spin, while the u of the K* mesons is not the same as the one of the kaons and it does vary with the spin. Assuming that this pattern applies also to the D and D* mesons, a tentative u-scan is performed with the 5 charged D and D* states, and a very good alignment is obtained with the exclusion of the D*(2)(2460). The LS fit with 4 states sets u at 34.67±0.016 MeV, with a p-value of 0.997. This alignment is spectacular, and would be compatible with the assumption that the D*(2010) and the D*(2640) are actually D mesons and that for the D family u does not vary with the spin. The fit of the three neutral states (excluding the D*(2)(2460)) gives u = 34.58 ±0.023 and a p-value of 0.960 (plots not shown).

meson type = D and D*										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u**	**dm**	**dm/m**
D	4	+	0		54	1869.3	0.5	34.617	-2.8	-0.15%
D*(2010)	4	+	1?		58	2010.0	0.5	34.655	-0.7	-0.04%
D(1)(2420)	3	+	1?		70	2427.0	5.0	34.671	0.3	0.01%
D*(2)(2460)	4	+	2		70	2459.0	4.0	35.129	32.3	1.31%
D*(2640)	3	+	?		76	2637.0	8.0	34.697	2.3	0.09%
D	4	0	0		54	1864.5	0.5	34.528	-2.9	-0.16%
D*(2007)	4	0	1?		58	2006.7	0.5	34.598	1.0	0.05%
D(1)(2420)	4	0	1?		70	2422.2	1.8	34.603	1.5	0.06%
D*(2)(2460)	4	0	2		70	2458.9	2	35.127	38.2	1.55%

summary D+ mesons	
u	35.67 ± 0.016
p-value	0.997
spin dependence	no
omitted	see text

16a

16b

16c

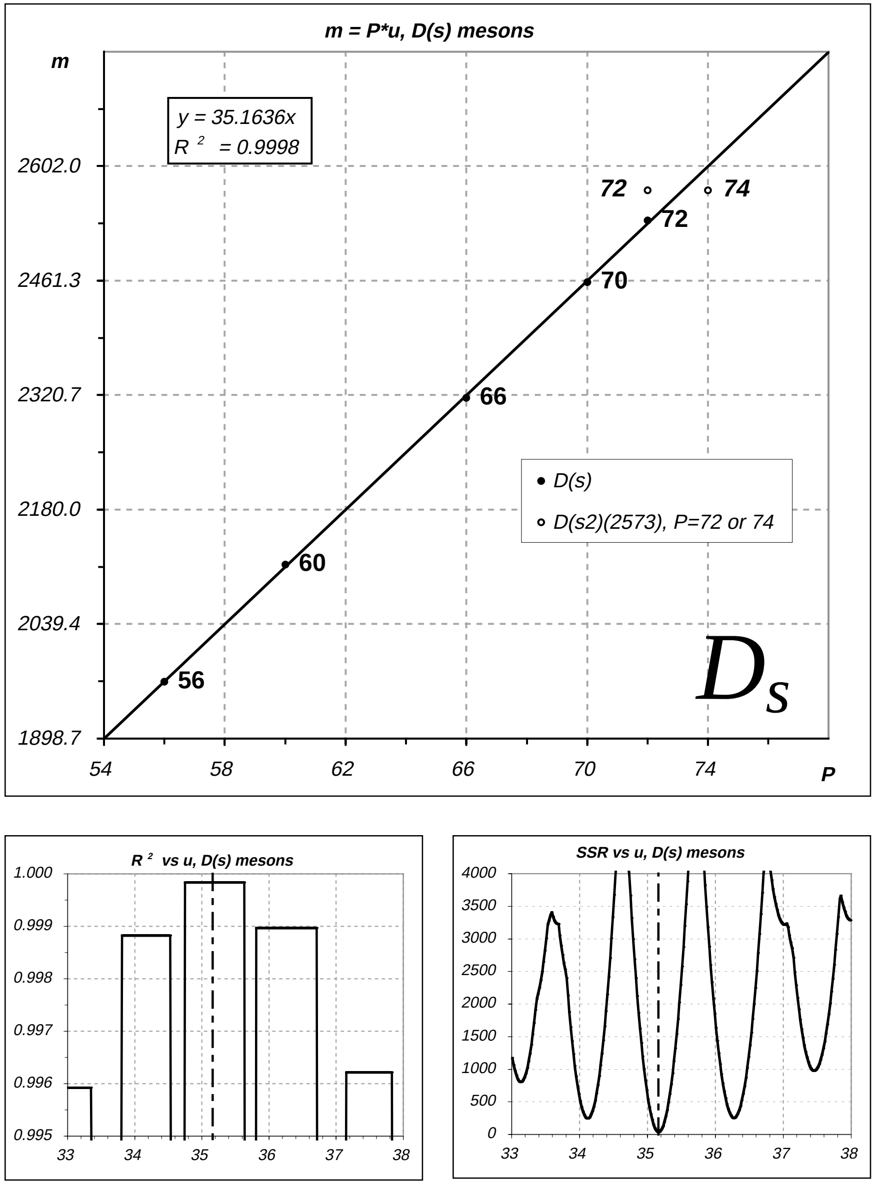


Fig. 16. Mass multiplicity, D(s) mesons: 16a, m vs P and line fit, ad-hoc mass scale; 16b, R^2 vs u; 16c, SSR vs u;

4.13 The D(s) and D(s)* mesons

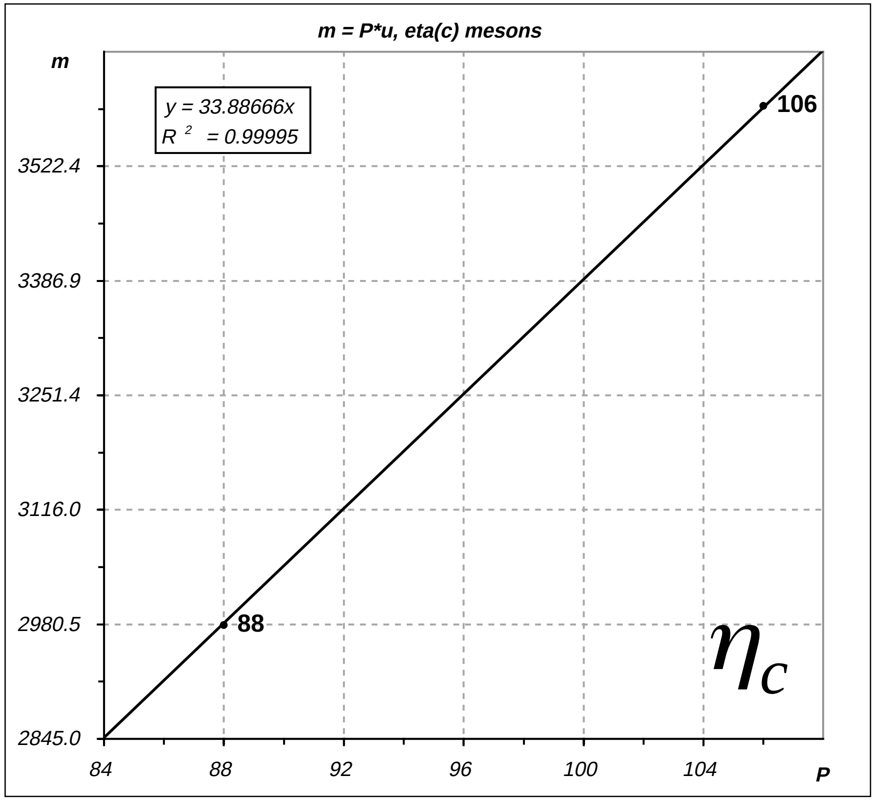
The D(s) meson are another intriguing combination of sharp multiplicity and apparent inconsistency with the quantum numbers listed by the PDG. With the same strategy used for the D and D*states, a u-scan is performed with all the 6 D(s) and D*(s) mesons, and the result is that 5 of them (including two D*(s)) can be aligned very sharply, while the D(s2)(2573) has a much larger residual. The LS fit finds u = 35.16 ±0.020 with a p-value of 0.997. The odds that this is by chance are really negligible, still the inconsistency with the quantum numbers is disturbing.

meson type = D(s) and D*(s)										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u**	**dm**	**dm/m**
D(s)	4	+	0		56	1968.5	0.6	35.152	-0.7	-0.04%
D(s)*	4	+	1?		60	2112.4	0.7	35.207	2.6	0.12%
D(sJ)*(2317)	4	+	0?		66	2317.4	0.9	35.106	-3.4	-0.17%
D(sJ)(2460)	4	+	?		70	2459.3	1.3	35.143	-2.2	-0.06%
D(s1)(2536)	4	+	1?		72	2535.4	0.6	35.213	3.6	0.14%
D(s2)(2573)	4	+	2?	3	74	2572.4	1.5	34.762	-29.7	1.58%

summary D(s) mesons	
u	35.16 ± 0.020
p-value	0.997
spin dependence	no
omitted	see text

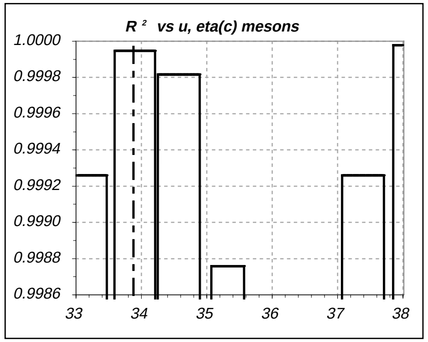
4.14 The eta(c) mesons

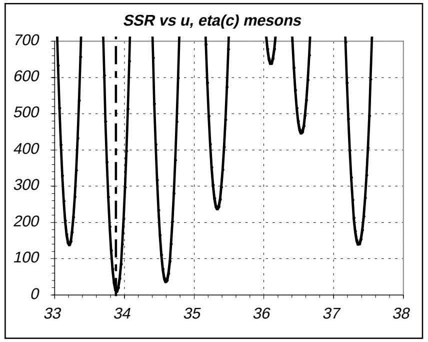
Only two eta(c) states are known, both with spin = 0, and the errors on the masses are just a few MeV, so that a u-scan can be tried. The result of the scan + fit procedure is u = 33.89 ±0.022 MeV. With just a couple of states it is difficult to go beyond an educated guess, still it is interesting that the mass unit for the eta(c) mesons is very close to the one of the etas = 33.86. The value J=0 for the eta(c)(2S) is a quark model prediction.



17a

17b



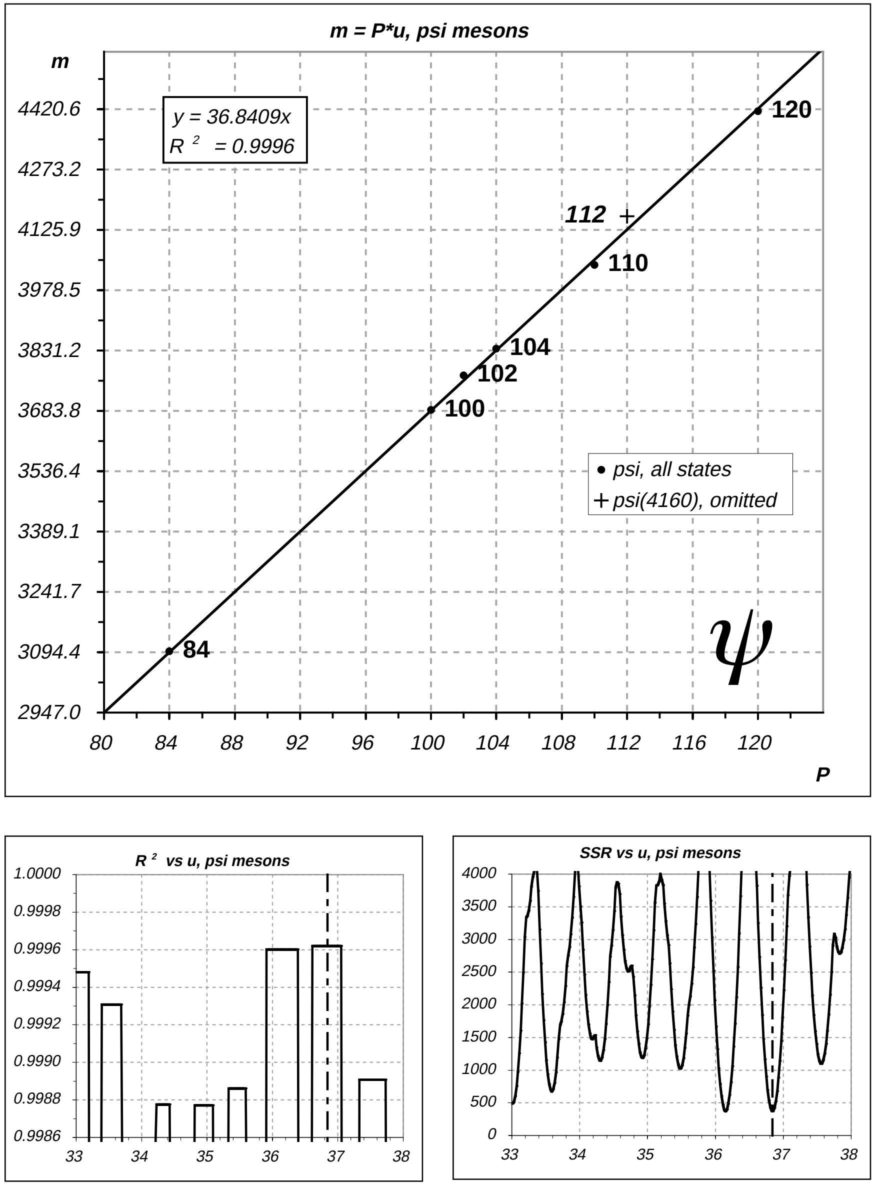


17c

Fig. 17. Mass multiplicity, eta(c) mesons: 17a, m vs P and line fit, ad-hoc mass scale; 17b, R^2 vs u; 17c, SSR vs u;

meson type = eta(c)										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u**	**dm**	**dm/m**
eta(c)(1S)	4	0	0		88	2979.6	1.2	33.859	-2.4	-0.08%
eta(c)(2S)	3	0	0		106	3594.0	5.0	33.906	2.0	0.05%

summary eta(c) mesons	
u	33.89 ± 0.022
p-value	not estimated
spin dependence	not assessed
omitted	none



18a

18b

18c

Fig. 18. Mass multiplicity, psi mesons: 18a, m vs P and line fit, ad-hoc mass scale; 18b, R^2 vs u; 18c, SSR vs u;

4.15 The psi mesons

The u-scan and fit on the 7 J/psi and psi states listed by the PDG show a good multiplicity alignment with an average residual of 9 MeV, apart from the psi(4160) with a residual of 33, rejected by Chauvenet's criterion. Its mass quoted by the PDG is based on a single measurement by DASP, and in the DASP paper the result of their analysis is compared with MARK1 data showing a more complex peak structure. Above the psi(4040) the MARK1 data show a peak at around 4110 and possibly more. The psi(4415) is seen unambiguously by both experiments. The DASP view of the discrepancy is: "..our data are in closer agreement with those of SLAC-LBL but show some differences in the finer details of the energy dependence. For instance the 4.16 structure is not resolved in the SLAC-LBL data". For sure there are differences, but the DASP interpretation is questionable. Apparently some MARK1 peaks were never identified or never made it to the PDG. A possible interpretation of their spectrum around 4100 that would fit the present scheme is: psi(4040), P=110; psi(4125), P=112; possibly a psi(4200) , P=114; no psi(4160).

The psi(3836) at P=104 is sharp on the line with a very small residual, and it is the only known J=2 psi meson. The analysis by the E705 experiment favors $J^P = 2^-$ while the PDG considers that quantum numbers are not established. If J=2 were to be confirmed, this would imply that in the psi family u may not depend on J as it does for other PC= - - families such as the phi and the omega. At any rate for a thorough spin analysis more states are needed. Omitting the psi(3836) from the sample does not change the value of u.

meson type = psi										
name	*	q	J	x	P	m	errm	u	dm	dm/m
psi(1S)	4	0	1		84	3096.9	4.0E-02	36.868	2.2	0.07%
psi(2S)	4	0	1		100	3686.0	9.0E-02	36.860	1.9	0.05%
psi(3770)	4	0	1		102	3769.9	2.5	36.960	12.1	0.32%
psi(3836)	3	0	2?		104	3836.0	13.0	36.885	4.5	0.12%
psi(4040)	4	0	1		110	4040.0	10.0	36.727	-12.5	-0.31%
psi(4415)	4	0	1		120	4415.0	6.0	36.792	-5.9	-0.13%
psi(4160)	4?	0	1	3	112	4159.0	20.0	37.134	32.8	0.79%

summary psi mesons	
u	36.84 ± 0.034
p-value	0.959
spin dependence	apparently no, but not really assessed
omitted	1 Chauvenet

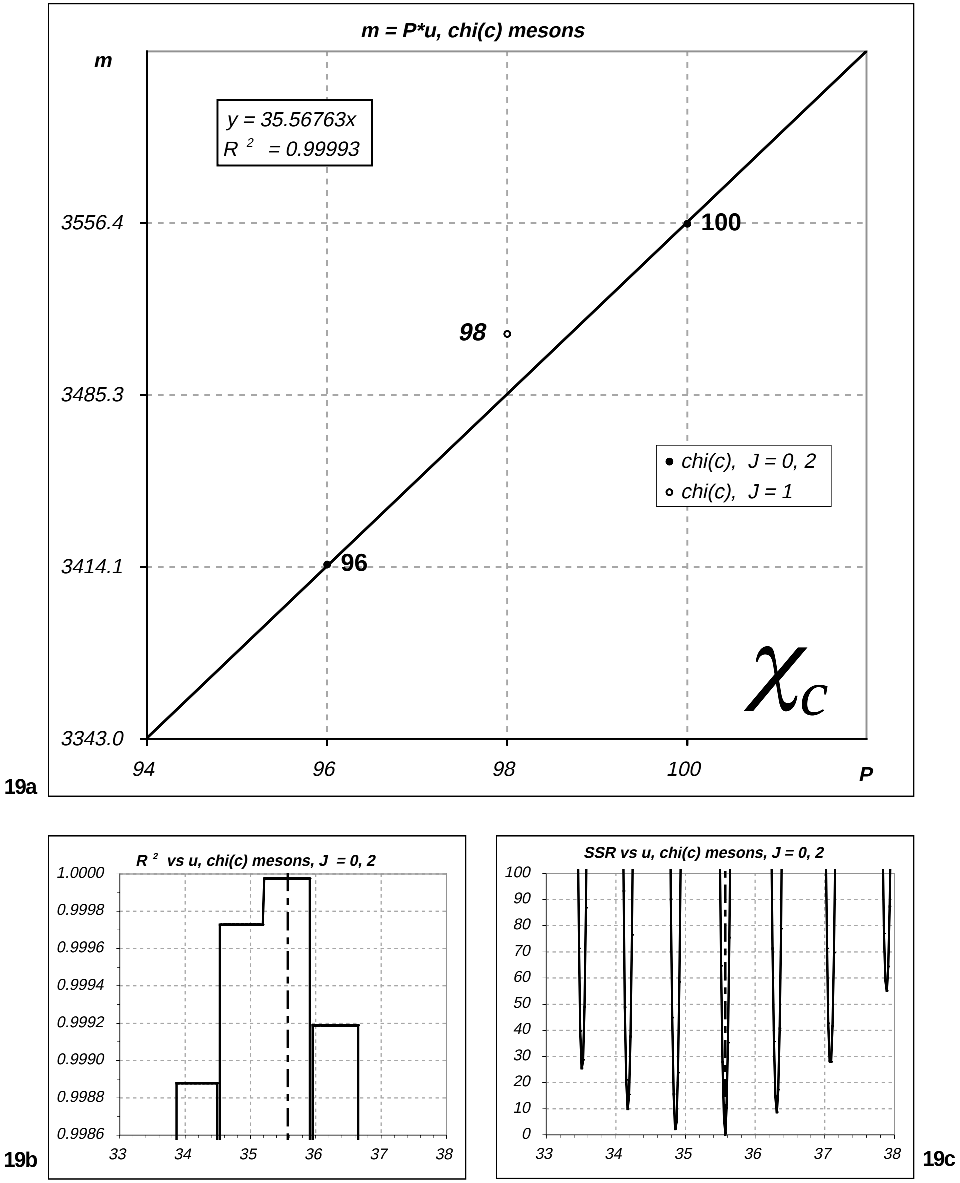


Fig. 19. Mass multiplicity, chi(c) mesons: 19a, m vs P and line fit, ad-hoc mass scale; 19b, R^2 vs u; 19c, SSR vs u;

4.16 The chi(c) mesons

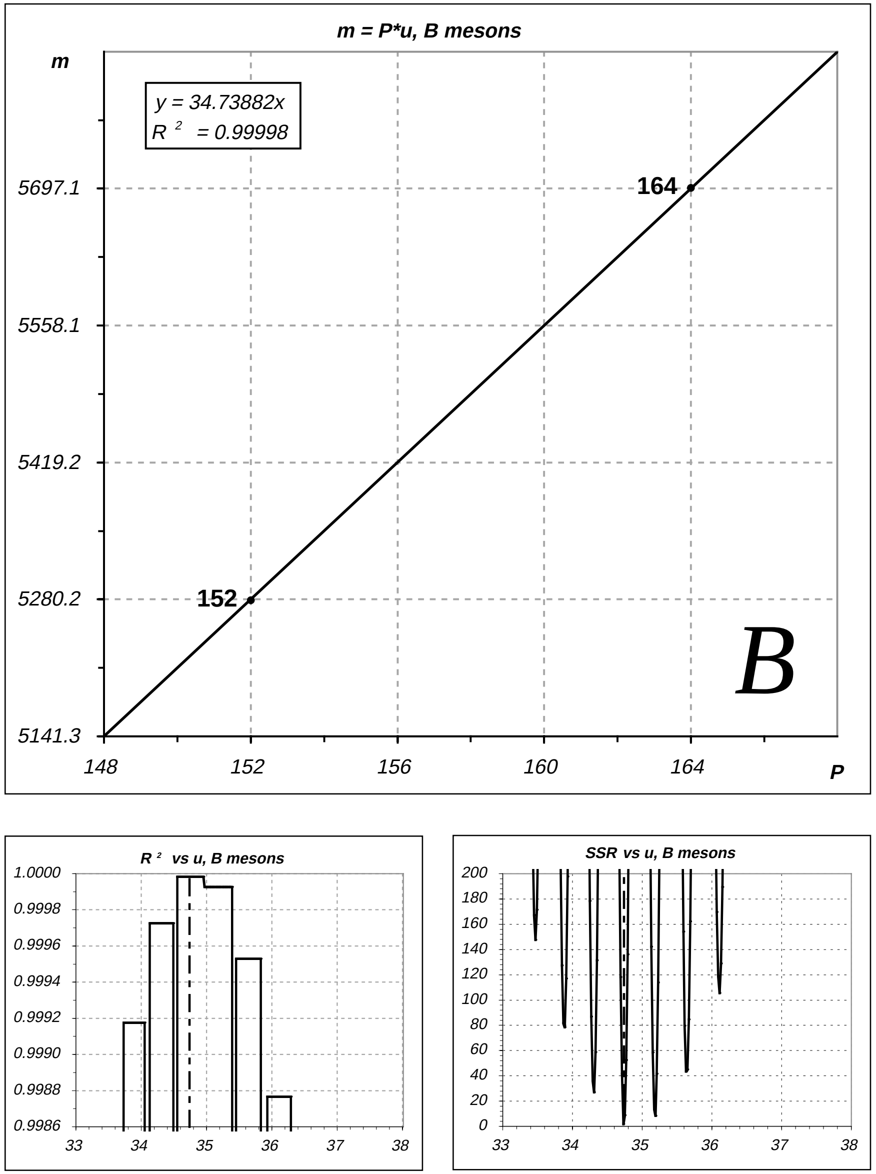
The three chi(c) states are very close in mass, and the errors on the masses are less that 1 MeV. Comparing the mass differences it appears that the (0,1) alignment with dm=45.7 MeV or the (1,2) with dm=95.4 are incompatible with a mass unit around 35 MeV, while the (0,2) with dm=141.1 is more promising. A u-scan with the three states produces P_i = 96, 98, 100, u=35.65, but a very poor R^2 of 0.962. By trying all 3 combinations of the chi(c) states two by two, the (0,2) combination shows a sharp alignment with R^2=0.99993, while the other two are poor: 0.9397 and 0.9128 . There clearly is a spin dependence, or the chi(c1) would also fit. It appears however that the two states with J=0 and J=2 may correspond to the same value of u, given their very precise and statistically consistent alignment (unless the spin of the chi(c0) or the chi(c2) were actually different from their current PDG assignment).

meson type = chi(c)										
name	*	q	J	x	P	m	errm	u	dm	dm/m
chi(c0)(1P)	4	0	0		96	3415.1	0.80	35.574	0.6	0.018%
chi(c2)(1P)	4	0	2		100	3556.2	0.13	35.562	-0.6	-0.016%
chi(c1)(1P)	4	0	1		98	3510.5	0.12	35.822	24.9	0.709%

summary chi(c) mesons	
u	35.57 ± 0.006
p-value	not evaluated
spin dependence	yes
omitted	none

4.17 The B and B* mesons

Only 3 B and B* states are listed by the PDG: the B (neutral and charged), the B* and the B*(J)(5732), a.k.a. B**. The mass difference (B*-B) is 45.8 MeV, such that these two particles cannot be part of the same multiplicity scheme. This is to be expected if the B* really is a B*, by analogy with the K and D mesons. As to the B**, by pairing it separately with the B and the B* it appears that it does not match with the B* but fits well with the B, for a value of u=34.74 ±0.005 MeV. With only one state, the B* family cannot be analyzed. The value of P listed for the B* is the closest match to the B alignment, just to



20a

20b

20c

Fig. 20. Mass multiplicity, B mesons: 20a, m vs P and line fit, ad-hoc mass scale; 20b, R^2 vs u; 20c, SSR vs u;

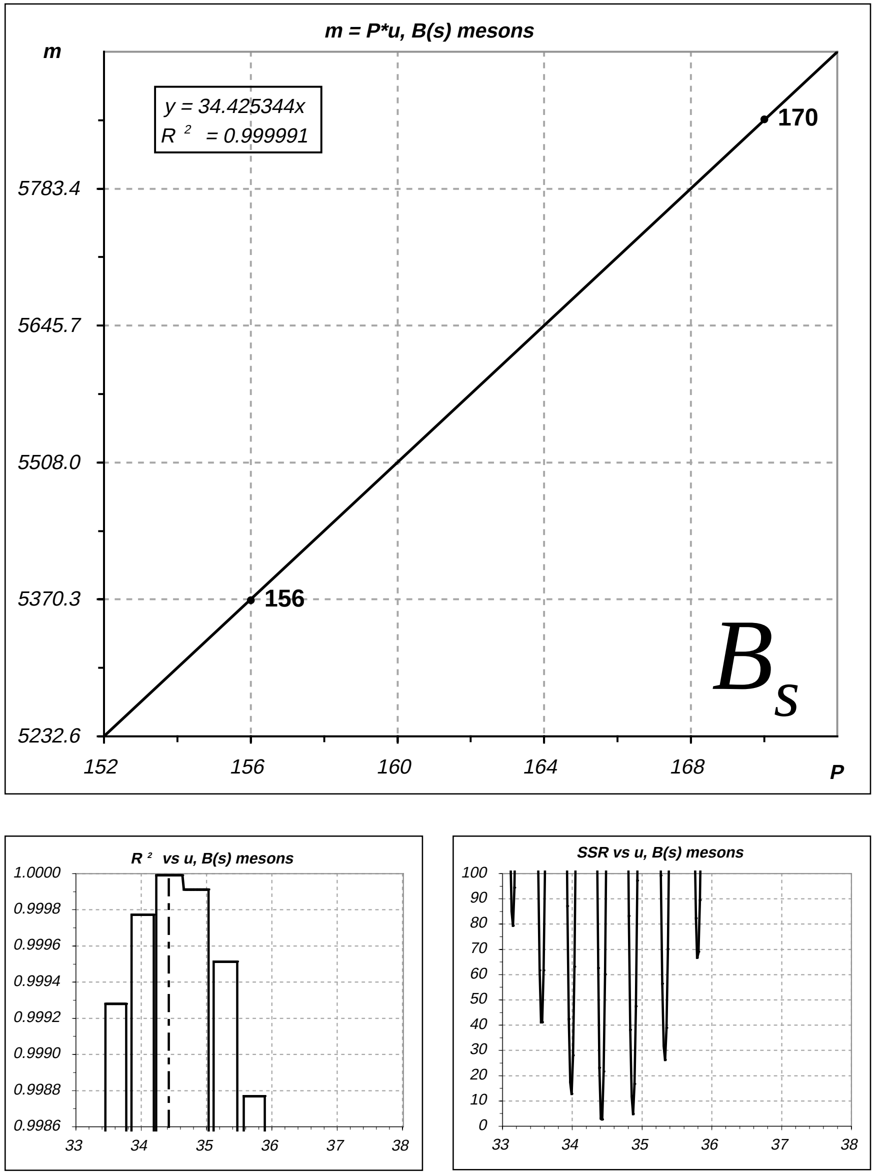
meson type = B										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u**	**dm**	**dm/m**
B	4	+	0		152	5279.0	0.5	34.730	-1.3	-0.02%
B	4	0	0		152	5279.4	0.5	34.733	-0.9	-0.02%
B(J)*(5732)	3		?		164	5698.0	8.0	34.744	0.8	0.01%
B*	4	0,+	1		154	5325.0	0.6	34.578	-24.8	-0.47%

summary B mesons	
u	34.74 ± 0.005
p-value	not estimated
spin dependence	not assessed
omitted	none

4.18 The B(s) and B(s)* mesons

The 3 known B(s) and B(s)* mesons do not align, given the B(s)*-B(s) mass difference of 47 MeV. By trying them out with a u-scan two at a time, the B(sJ)* combines with the B(s) better than with the B(s)*, with u = 34.42 ±0.004. The value of P listed for the B(s)* is the closest match to the B(s) alignment.

There are not enough B, B, B(s) and B(s)* states in order to perform a significant analysis, and there seem to be ambiguities in the assignment of the quantum numbers. The B and B(s) results are nothing more than an educated guess.*



21a

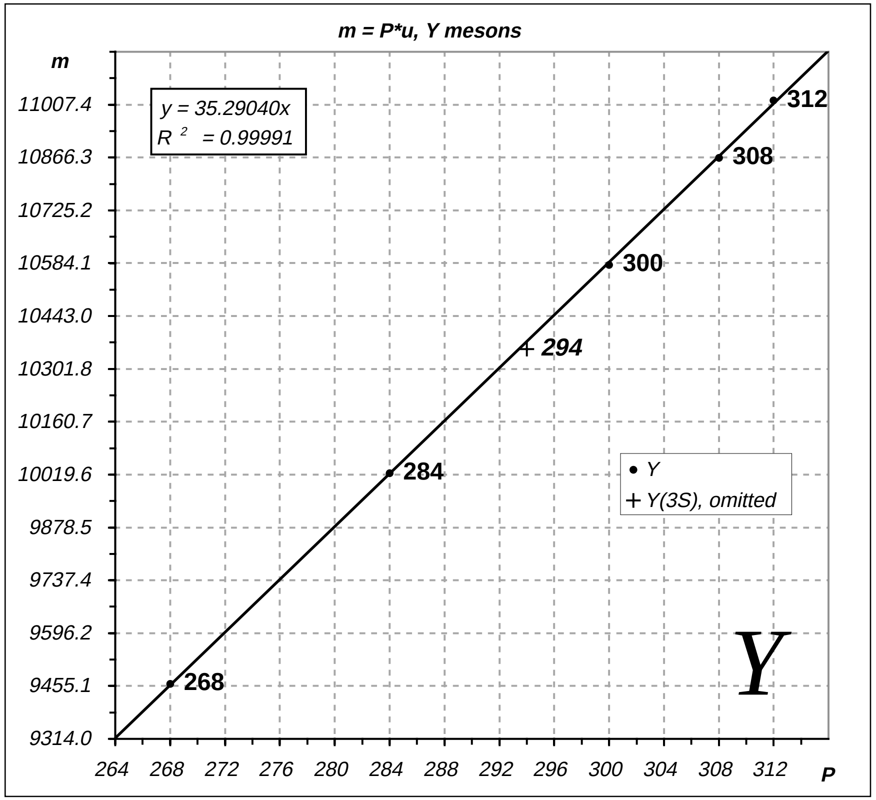
21b

21c

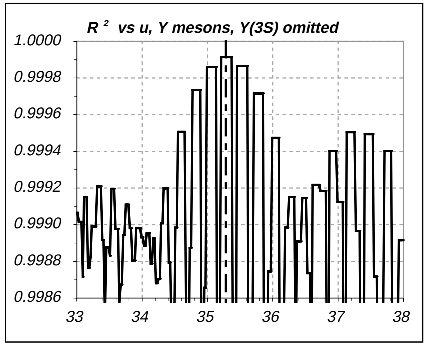
Fig. 21. Mass multiplicity, B(s) mesons: 21a, m vs P and line fit, ad-hoc mass scale; 21b, R^2 vs u; 21c, SSR vs u;

meson type = B(s) and B(s)*										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u**	**dm**	**dm/m**
B(s)	4	0	0		156	5369.6	2.4	34.421	-0.8	-0.01%
B(sJ)*	3		?		170	5853.0	15.0	34.429	0.7	0.01%
B(s)*	3	0	1?		158	5416.6	3.5	34.282	-22.6	-0.42%

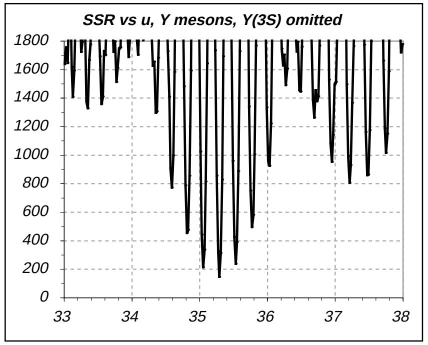
summary B(s) mesons	
u	34.42 ± 0.004
p-value	not estimated
spin dependence	not assessed
omitted	none



22a



22b



22c

Fig. 22. Mass multiplicity, Y mesons: 22a, m vs P and line fit, ad-hoc mass scale; 22b, R^2 vs u; 22c, SSR vs u;

4.19 The Upsilon mesons and other b-bbar families

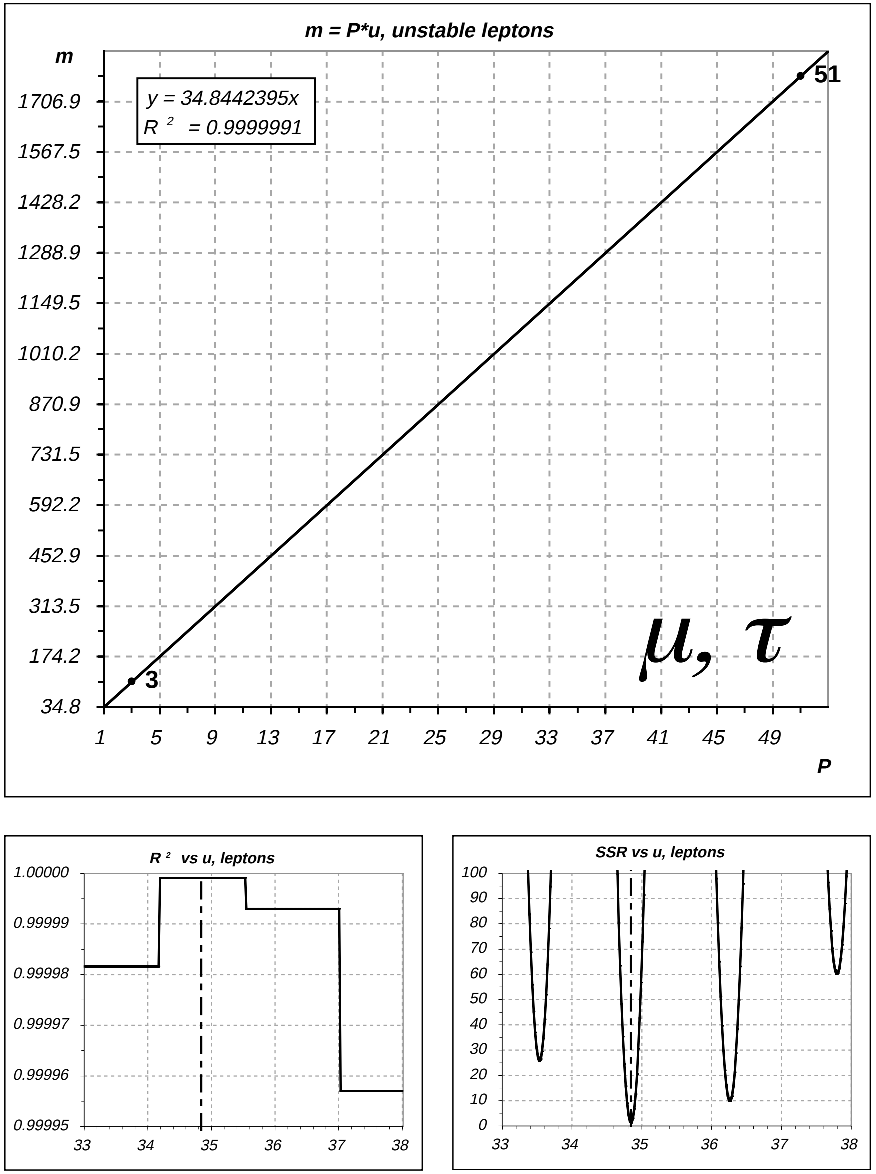
A u-scan of the 6 Υ mesons shows a very good alignment at u = 35.28 MeV. The Υ(3S) has a residual of 20 MeV and can be rejected by Chauvenet's criterion. By omitting the Υ(3S) the p-value improves from 0.963 to 0.985 and the behavior of the SSR in the u-scan gets considerably sharper, with a minor change in the result of the fit.

The mass of the Υ(3S) quoted by the PDG is the result of a single measurement at VEPP-4 with the MD-1 detector in 2000, confirming the value from an earlier measurement. In the same paper also the masses of the Υ(1S) and Υ(2S) are reported, measured in the same apparatus. A 1999 paper by the CLEO collaboration analyzed the decays Υ(3S) -> p p Υ(1S) and -> p p Υ(2S), and reported anomalies that they were unable to understand theoretically. The same channels were studied with the CUSB-II detector at CESR and published in 1992. The conclusion is the same: "the di-pion mass spectrum cannot be explained by the current theoretical models in a satisfactory way".

meson type = Y										
name	*	q	J	x	P	m	errm	u=m/P	dm	dm/m
Y(1S)	4	0	1		268	9460.3	0.26	35.300	5.5	0.06%
Y(2S)	4	0	1		284	10023.3	0.31	35.293	4.0	0.04%
Y(3S)	4	0	1	3	294	10355.2	0.50	35.222	-16.8	-0.16%
Y(4S)	4	0	1		300	10580.0	3.50	35.267	-3.7	-0.04%
Y(10860)	4	0	1		308	10865.0	8.00	35.276	-0.9	-0.01%
Y(11020)	4	0	1		312	11019.0	8.00	35.317	11.9	0.11%

summary Y mesons	
u	35.29 ± 0.009
p-value	0.985
spin dependence	not assessed, all states are J=1
omitted	1 Chauvenet

The other b-bbar meson families cannot be analyzed for multiplicity: there is only one eta(b), and the chi(b) family is difficult. The u-scan shows a poor alignment of the 6 states, suggesting a dependence of u from the spin. This is confirmed by the mass differences among the three (1P) states, and the same is true for the (2P) states. Separate u-scans for J=0,1 and 2 with two states each do not constrain u unambiguously.



23a

23b

23c

Fig. 23. Mass multiplicity, mu and tau leptons: 23a, m vs P and line fit, ad-hoc mass scale; 23b, R^2 vs u; 23c, SSR vs u;

4.20 The unstable leptons

particle type = unstable lepton										
name	*****	**q**	**J**	**x**	**P**	**m**	**errm**	**u=m/P**	**dm**	**dm/m**
mu	4	-	1/2		3	105.66	5.0E-06	35.219	1.13	1.065%
tau	4	-	1/2		51	1776.99	0.275	34.843	-0.07	-0.004%

As indicated in section 1 the mass multiplicity hypothesis applies to mesons, baryons and leptons. The baryon mass analysis will be published separately, while the unstable leptons are presented here.

The muon and the tau lepton can be analyzed as a family with the same procedure used for the mesons, but for the multiplicity P_i being odd rather than even. The u-scan identifies the best alignment at P=3 for the muon and P=51 for the tau meson. It corresponds to u = 34.84 ±0.022, with R^2 = 0.9999991. Spin dependence is not an issue, and the p-value of the hypothesis is not evaluated.

Summary of mass unit analysis, mesons

							states	omitted					used	
type	k	u	erru	uw	p-value	du/dJ	PDG	(1)	(2)	(3)	(4)	tot		rating
pi	**3**	34.69	0.051	34.68	0.997	N	11	1		1	1	3	8	****
b	**9**	36.16	0.050	36.16	0.990	N	3						3	***
rho	**6**	35.19	0.071	**35.31**	0.973	N	11	2		1		3	8	***
a					0.995	Y	13	2				2	11	****
a(0)	**5**	35.00	0.073	**35.17**	0.941									
K	**6**	35.34	0.073	35.39	0.943	N	11				1	1	10	***
K*					0.882	Y	12	1		1	2	4	8	
K*(1)	**2**	34.35	0.016	34.35										****
eta	**0**	33.86	0.053	33.86	0.999	N	13	4				4	9	****
h	**2**	34.42	0.056	34.43	0.975	N	6	2				2	4	****
omega					0.934	Y	7	1				1	6	****
omega(1)	**8**	35.80	0.049	35.81	0.943									
phi					0.732	Y	3						3	**
phi(1)	**10**	36.51	0.050	**36.41**										
f	**7**	35.78	0.070	**35.60**	0.998	?	33	5	18			23	10	***
D^+	**3**	34.67	0.016	34.66	0.997	N	5						5	****
D^0		34.58	0.023	34.60	0.960	N	4						4	****
D(s)	**5**	35.16	0.021	35.15	0.997	N	6						6	****
eta(c)	**0**	33.89	0.022	33.87			2						2	**
psi	**12**	36.84	0.034	36.87	0.959		7			1		1	6	****
chi(c)	**7**	35.57	0.006	35.56		Y	3						3	**
B	**3**	34.74	0.005	34.73			3				1	1	2	*
B(s)	**2**	34.42	0.004	34.42			2						2	*
Y	**6**	35.29	0.009	35.30	0.985		6			1		1	5	****
		avg->	0.044		0.949		161	18	18	5	5	46	115	**<-tot**

leptons	**4**	34.84	0.022	34.84			2						2	**

Fig. 24. Summary table of mass multiplicity analysis by meson family:
u is the mass unit from the LS fit (no weights) in MeV, and erru is the error on u;
uw is the mass unit computed with a weighted fit, boldface if significantly different from u;
the p-value of the multiplicity hypothesis is not evaluated for samples of 2 mesons only;
states can be removed from the sample for various reasons:

1. errm > 30 MeV
2. PDG listing ambiguity
3. large residual, rejected by Chauvenet's criterion
4. averaging of I-multiplet values, e.g. p^+ and p^0

The parameter k will be defined in 5.3, the star rating is subjective by this author.

5. Summary

The table at figure 24 summarizes the results of the meson mass analysis.

5.1 Sample and statistics

All states listed by the PDG have been considered, including the light unflavored mesons measured by only one group and the "non q-qbar" states ; out of a total of 161 mesons, about 10% have been discarded a priori because of a large measurement error on the mass, 5 have been averaged with their I-multiplet companion state, and 5 rejected by Chauvenet's criterion. 18 f mesons have been omitted to obtain a reduced cleaner sample (with those f states included the p-value is not as good, u is spin dependent and the value of u for the f(0) is 35.54 ±0.078).

The multiplicities P_i are identified unambiguously for all families by the u-scan. Because of the constraint at the origin, this procedure is effective also for samples of only two states, especially if the errm are small.

5.2 Results

14 of the 19 families have more than two states, and for them the mass multiplicity hypothesis is verified with an average p-value of 0.95. For some families a dependence of u from the spin has been determined, for many others it can be excluded, while there are a few dubious cases. For samples of 2 states only it is not clear how to compute the p-value, while for families where u is spin dependent a more refined analysis could produce marginally better p-values.

Overall the mass-multiplicity-by-family hypothesis appears to be valid, and for some of the families with many states and no spin dependence, such as the pions or the etas, the alignment is impressive with a p-value of 0.99 or better. Kaons and f mesons are not as sharp, while for families with very few states there are problems due to identification. Sometimes the PDG quantum numbers are not measured but derived from quark model predictions, still if the masses are measured very precisely and there is no spin dependence (e.g. with the D and D(s) mesons) the multiplicity is so constraining that it provides consistency checks on the quantum numbers.

The values of the mass unit u range from 33.86 for the eta to 36.84 for the psi. By inspecting the values of u in table 24 various regularities become apparent:

- the eta and eta(c) are very close and at the low end of the u spectrum;
- the q-qbar symmetric vector and scalar mesons are in the upper range;
- the q-qbar asymmetric states are in the lower range, apart from the b.

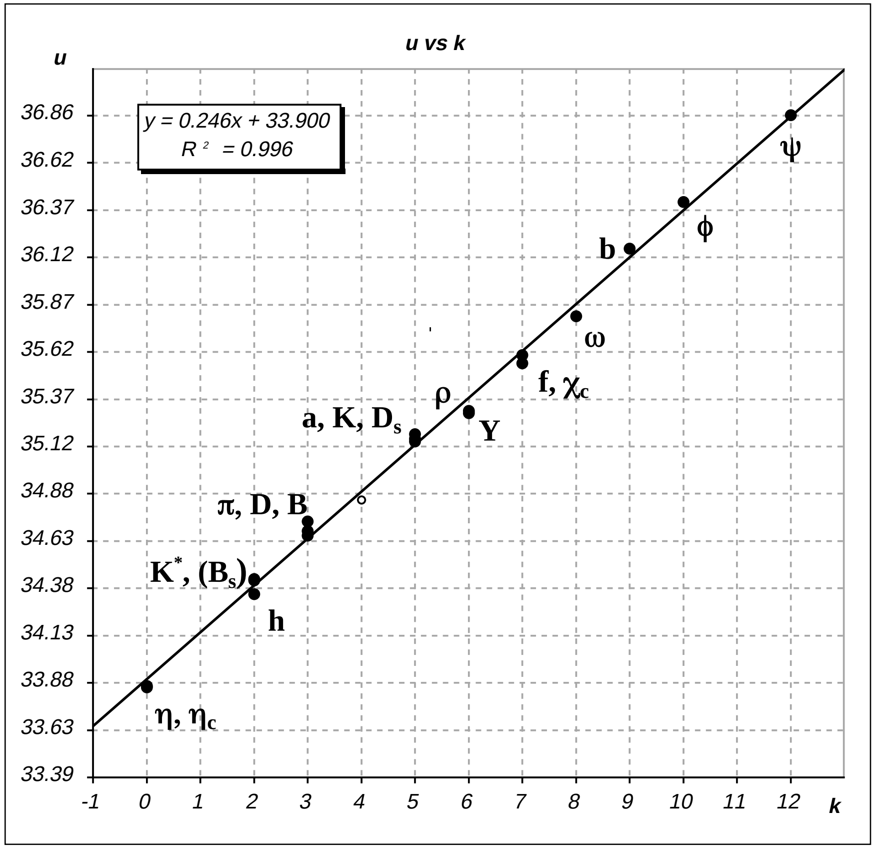


Fig. 25. u quantization $u = u_o + k*du$, k=0,1,..12 ; the labels above the line refer to the q-qbar asymmetric mesons, while the q-qbar symmetric are listed below the line; the open point at k=4 corresponds to the leptons; the radius of the points is approximately equal to the average error on u; in the plot the value for the kaons is u=35.19 (only 4-star states) rather than the one of the table at figure 24.

5.3 Is the mass unit quantized?

Other values of u in table 24 seem to be recurrent, and by sorting the uw column and assigning to each value an integer number proportional to the distance from u(eta), the u quantization pattern of figure 25 is obtained. The 19 values of uw are distributed on a grid u(k) of 12 equal intervals of about 0.25 MeV, with k ranging from 0 to 12, and the p-value of this u quantization hypothesis, computed by Montecarlo simulation on the basis of the R^2 of the line fit of figure 25, is equal to 0.95. On this u-grid the regularities become more specific:

q-qbar symmetric states

- $J^{PC} = 0^{-+}, 2^{-+}$ .. k even and < 6 : eta and eta(c) at k=0
- $J^{PC} = 1^{+-}, 3^{+-}$ .. k even and < 6 : h at k=2;
- $J^{PC} = 1^{--}, 2^{--}$ .. k even and ≥ 6 : Y, omega, phi, psi at k=6, 8, 10, 12;
- $J^{PC} = 0^{++}, 1^{++}$ .. k odd and > 6 : f and chi(c) at k=7

q-qbar asymmetric states

if two assignments are modified, by moving the K from 6 to 5 and neglecting the B(s), then the following rules apply:

- $J^{PC} = 0^{-+}, 2^{-+}$ .. k odd and < 6 : pi, D, B at k=3
- $J^{PC} = 1^{+-}, 3^{+-}$ .. k odd and > 6 : b at k=9
- $J^{PC} = 1^{--}, 2^{--}$ .. k even and ≤ 6: K*at k=2, rho at k=6
- $J^{PC} = 0^{++}, 1^{++}$ .. k odd and < 6 : a(0) at k=5;

with a remarkable symmetry pattern between corresponding rules of the two categories. Actually the K family fit is not as clean as other pseudoscalars, and by using only 4-star kaons the mass unit equals 35.19 (k=5), while the u values for the B and B(s) are just an educated guess given the small number of states and suspected quantum numbers ambiguities.

5.4 Conclusions

The 35 MeV mass quantum for particles has been proposed by several authors but it is seldom quoted and basically ignored. The comprehensive and more precise mass multiplicity patterns described in this article are obtained with a straightforward statistical analysis of the complete meson mass spectrum, and strengthen the mass quantization hypothesis considerably. The selective dependence of u from the spin and the second-order effect u(k) with its amazing regularities are new results, and it is hard to imagine that they might be an artifact or just an improbable coincidence.

A physics interpretation is being developed and will be published separately.

Appendix A. Statistical analysis

Particle masses and their errors have been taken from the PDG RPP from year 2002 [8,9], adding the two D(s) states discovered in 2003. The analysis is programmed in FORTRAN, HBOOK and PAW and the resulting plots are charted with MS Excel.

A.1 R^2

The present analysis is based on least squares straight line fits of mass versus P. The correlation coefficient R^2 is the square of Pearson's product moment correlation R:

$$R = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\mathrm{sqrt}((n(\Sigma x^2) - (\Sigma x)^2)(n(\Sigma y^2) - (\Sigma y)^2))}$$

and expresses the fraction of the total variation of the sample that is accounted for by the least-squares regression line. It varies from 1 (perfect linear correlation) to 0 (no correlation).

A.2 Assignment of P_i and fit of u

The hypothesis to be tested is that for all mesons of the same family the mass can be expressed as $m_i=u*P_i$, u being a constant in the vicinity of 35 MeV and the P_i even integers. For the meson families comprising one or more low mass states, the assignment of the P_i can be made by trial and error, dividing the mass by the value of u for the state of that family present in table 1a. To confirm the values, and to assign the P_i for heavier families, a u-scan is performed in a range around 35 MeV.

Varying u in small steps, the P_i are derived from the masses for all the mesons in the family, and R^2 is computed along with the sum of the squares of the residuals (SSR). The set of P_i with the maximal R^2 is retained, and the minimum of the SSR corresponds to an approximate value of u. The value of u and its error is then computed by a non-weighted least squares fit m vs P. The mass unit is then recomputed with a weighted LS fit.

A.3 Fit stability

If the number of states in the sample permits it, the fit is tested for non-linearity at the low mass end by removing points one by one starting from the lowest mass and refitting.

A.4 Goodness-of-fit and p-value of the hypothesis

The level of agreement between the data and the hypothesis is quantified by taking R^2 as goodness-of-fit statistics, with $R^2 = 1$ corresponding to the best level of agreement. Following the PDG terminology, the agreement is expressed by the p-value, defined as the probability to find the R^2 in the region of equal or lesser compatibility with the hypothesis (sometimes the p-value is improperly referred to as "confidence level").

In the present case R^2 is close to 1 by construction, being the P_i chosen as the best set of even integers fitting the experimental masses for a given value of the mass unit. The p-value is computed case by case with a Montecarlo simulation tailored to the value of u, the count of the sample and the mass range.

When the sample consists of only 2 states the procedure is not really sound and it is not applied.

A.5 Spin dependence

A possible dependence of u from J within a given meson family can be assessed by computing u separately for subsamples of states with different values of the spin. Considering two samples of counts n_1 and n_2 with mass units u_1 and u_2 and standard deviations σ_1 and σ_2, for a level of significance α=0.01 the test statistic is computed as

$$Z = (u_1-u_2) / \mathrm{sqrt}(\sigma_1{}^2/n_1 + \sigma_2{}^2/n_2)$$

and the hypothesis $u_1 = u_2$ is rejected if Z is greater than $Z(\alpha/2)$ =2.58 in absolute value. For most of the families where u is spin-dependent the effect is unambiguous and corresponds to values of Z of order 10 or more.

A.6 Chauvenet's criterion for rejecting data points

Points of the sample that exhibit a large fit residual d compared to the standard deviation of the residuals σ can be rejected using Chauvenet's criterion. If the ratio d/σ is greater than a tabulated critical deviation cd, then the point can be discarded; cd is a function of N, the count of the sample, and varies from 1.38 for N=3 to 1.96 for N=10. The rejection procedure can be applied only once. When a point is discarded, the PDG listing and possibly the experimental papers are analyzed to try and determine the likely origin of the deviation that caused the rejection.

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