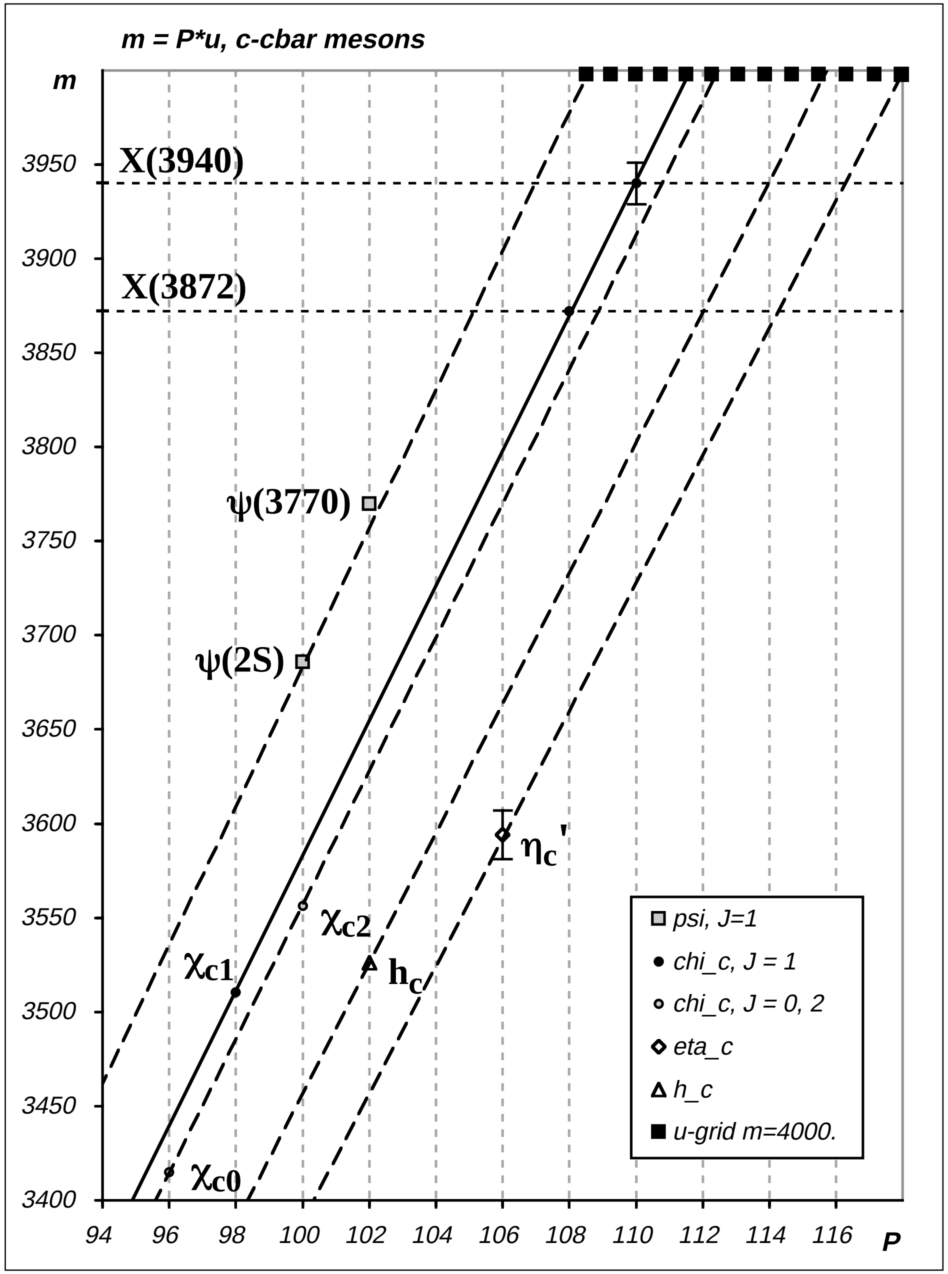

Preprint: p3a-2005-002
10-JUN-2005

Mass constraints and the identity of the X(3872) and X(3940) mesons

Paolo Palazzi

Abstract

With reference to a very precise but little-known mass quantization scheme, the masses of the X(3872) and X(3940) mesons are compared with the various c-cbar meson families. The only statistically significant and consistent match is with the chi_c(1). This result is in agreement with the experimental constraints on the possible J^{PC} quantum numbers of the X(3872), recently published by the Belle Collaboration, where the accumulated evidence strongly favors a $J^{PC} = 1^{++}$ assignment for the X(3872).

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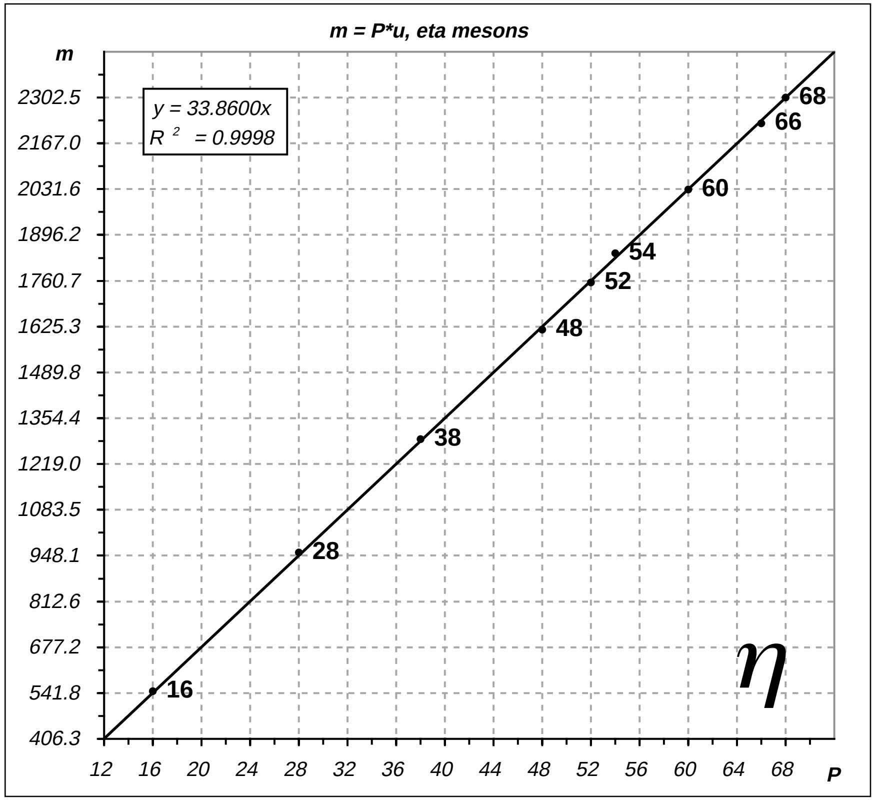
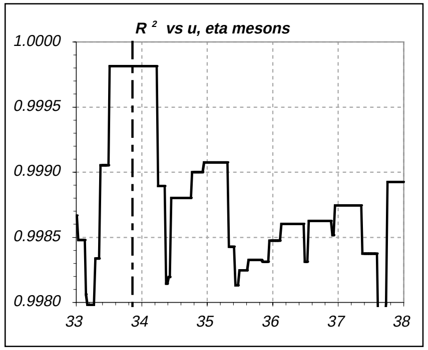
Table of Contents

Revision Record

10-JUN-2005 preprint, original electronic submission, published on the independent electronic Particle Physics Preprint Archive p3a.



1a



1b

meson type = eta									
name	*	q	J	x	P	m	errm	u=m/P	dm
eta	4	0	0		16	547.3	0.1	34.206	5.5
eta'(958)	4	0	0		28	957.8	0.1	34.206	9.7
eta(1295)	4	0	0		38	1293.0	5.0	34.026	6.3
eta(1440)	4	0	0	1	42	1435.0	35.0	34.167	12.9
eta(2)(1645)	3	0	2		48	1617.0	5.0	33.688	-8.3
eta(1760)	3	0	0		52	1756.0	11.0	33.769	-4.7
eta(2)(1870)	3	0	2		54	1842.0	8.0	34.111	13.6
eta(2)(2030)	2	0	2		60	2030.0	20.0	33.833	-1.6
eta(2190)	2	0	0	1	64	2190.0	50.0	34.219	23.0
eta(2)(2250)	2	0	2		66	2225.8	13.0	33.723	-9.0
eta(2225)	3	0	0	1	66	2227.0	35.0	33.742	-7.8
eta(2280)	3	0	0		68	2302.5	12.0	33.860	0.0
eta(4)(2320)	2	0	4	1	68	2328.0	38.0	34.235	25.5

Fig. 1. Meson mass multiplicity: 1a, mass number assignment P for all PDG eta mesons, excluding those with error > 30 Mev/c^2, and fit of the mass unit; 1b, R2 vs u from the u-scan to determine the mass numbers P; 3c, eta mesons table with excluded states grayed out. (reproduced from [1], the eta states table has been simplified).

1. Regularities in the meson mass spectrum

An analysis of the meson mass spectrum by the present author [1] has confirmed previous evidence of quantization of meson masses with a unit of about 70 MeV/c^2. This quantization is part of a broader scheme covering all hadrons and also the unstable leptons. The idea was proposed by Y. Nambu [2] in 1952, and extended by other authors (most notably by M. H. Mac Gregor) as new states were discovered. A historical account can be found in [1].

Nambu's original conjecture was that 1/2 of the π mass, about 70 MeV/c^2, is a significant mass unit. On this basis he then attributed to all known particles a "mass number", integer or half-odd: 0 for the photon, 3/2 for the μ, 2 for the π, 7 for the K, 27/2 for the nucleon and so on. Deviations from the rule are of the order of ± 1/10 of the mass unit, and in addition:

- the mass unit agrees with Heisenberg's natural unit m_e/α (= 70.02 MeV/c^2);
- bosons have integer, while fermions half-integer, mass number;
- the electron mass and the $\pi^{\pm}$–π^{o} mass difference are a kind of fine structure.

The quantization with a single mass unit is statistically relevant only below 1 GeV/c^2, and above that value it just blurs out. Actually the rule is valid throughout the mass spectrum, simply with *mass units that are function of the particle type*. This extended hypothesis has been probed for all mesons listed by the PDG, with the exception of those states with an error on the mass that is too large to permit the analysis [1].

To work with integer mass numbers, the mass unit used is half Nambu's one, 35 MeV/c^2, so that the mass numbers P are even for mesons and odd for baryons and unstable leptons. The statistical analysis, based on automatic procedures *and using no physics hypothesis*, shows that:

- $m_i = P_i * u_j$ [P even and u_j close to 35 MeV/c^2] for all known mesons, when grouped in families according to their PDG q-qbar assignment, I and J^{PC}, with goodness-of-fit indicator of the hypothesis (p-value) > 0.95; e.g. for the η family of figure 1a, u_j = 33.86 ± 0.053 with p-value. = 0.999;
- most vector and scalar families, but no pseudoscalars, show a variation of u with the spin, with du/dJ of about 0.25 MeV/c^2 per unit of spin;

Within each family, the mass values are quantized by $2*u_j$ and a state with an abnormally large fit residual can be rejected using a statistical indicator. In this way five states out of a total of 130 have been singled out, and the possible origin of the residual investigated. Apparent inconsistencies in the PDG quantum numbers of the D and D_s mesons have also been identified [1].

With the same criterion, the compatibility of unidentified states with established alignments of various candidate families can be checked. Before doing so for the X(3872) and X(3940) c-cbar mesons, in the next section a remarkable second quantization of the mass will be briefly outlined.

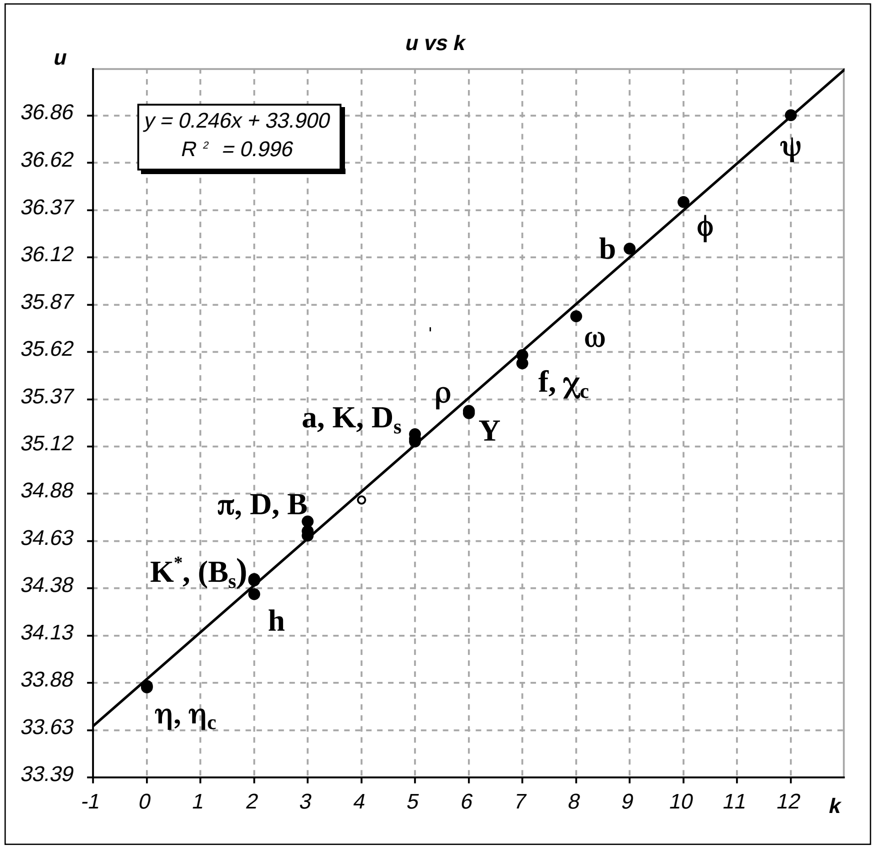


Fig. 2. u quantization $u = u_o + k*du$, k=0,1,..12 ; the labels above the line refer to the q-qbar asymmetric mesons, while the q-qbar symmetric are listed below the line; the open point at k=4 corresponds to the leptons; the radius of the points is comparable to the average error on u; in the plot the value for the kaons is u=35.19 (only 4-star states) rather than the one obtained with all the states; please refer to the text in section 2 for the rationale of this change.

2. Quantization of the mass unit

With the procedure described in section 1 a total of 17 meson families could be analyzed, and the corresponding mass unit computed with a precision of the order of 0.05 MeV/c^2, or better for high-mass families. An inspection of the set of the values of u for all the families indicates that some of them seem to be recurrent (table not shown here). By sorting them and assigning to each value an integer number proportional to the distance from u(eta), the u quantization pattern of figure 2 is revealed [1]. For the meson families with u dependent on the spin, the value of u corresponding to the base spin value is considered.

The 17 values of u are distributed on a grid u(k) of 12 equal intervals of about 0.25 MeV, with k ranging from 0 to 12. The p-value of this u quantization hypothesis, computed by Montecarlo simulation on the basis of the R^2 of the line fit of figure 2, is equal to 0.95. The u-grid obeys the following rules with respect to the quark composition and J^{PC}:

q-qbar symmetric states

- $J^{PC} = 0^{-+}$, 2^{-+} .. k even and < 6 : eta and et_c at k=0
- $J^{PC} = 1^{+-}$, 3^{+-} .. k even and < 6 : h at k=2;
- $J^{PC} = 1^{--}$, 2^{--} .. k even and ≥ 6 : Y, omega, phi, psi at k=6, 8, 10, 12;
- $J^{PC} = 0^{++}$, 1^{++} .. k odd and > 6 : f and chi_c at k=7

q-qbar asymmetric states

if one assignment is modified, by moving the K from 6 to 5, and the B_s point is neglected, then the following rules apply:

- $J^{PC} = 0^{-+}$, 2^{-+} .. k odd and < 6 : pi, D, B at k=3
- $J^{PC} = 1^{+-}$, 3^{+-} .. k odd and > 6 : b at k=9
- $J^{PC} = 1^{--}$, 2^{--} .. k even and ≤ 6: K*at k=2, rho at k=6
- $J^{PC} = 0^{++}$, 1^{++} .. k odd and < 6 : a(0) at k=5;

and the corresponding rules of the two categories are highly symmetric. Actually the K family fit is not as clean as other pseudoscalars, and by using only 4-star kaons the mass unit equals 35.19 (k=5), while the u values for the B and B_s are just an educated guess given the small number of states and suspected quantum numbers ambiguities. With just these two changes, the overall correlation pattern in the u-grid is really remarkable, and it is hard to imagine that these results might be an artifact or just an improbable coincidence.

In what follows the mass quantization properties summarized in this section and in the previous one will be used to tentatively match the X(3872) and X(3940) mesons with all c-cbar families.

3. The X(3872) and X(3940) c-cbar mesons

In 2003 physicists of the Belle experiment reported evidence of a narrow c-cbar meson X(3872) in the channel B --> X(3872)K --> J/ψππK [3], later seen also by CDF [4] and D0 [5] in inclusive proton-antiproton collisions, and by BaBar [6] in exclusive B-meson decays. The state is listed in the PDG 2004 with quantum numbers not established [7].

In 2004 another c-cbar state, X(3940) was detected by Belle [8]. The decay mode of this second state into ωJ/ψ, apparently inconsistent with the J/ψππ decay mode originally detected, has been branded Y(3940) [9], to indicate the possibility that it might be a different state with very close mass.

The discovery of the X(3872) with J^{PC} not directly measured has generated an animated discussion in the community, because of the seeming inconsistency with the current state of knowledge of the charmonium system and transitions. The absence of an obvious charmonium assignment has lead to suggestions of non-charmonium possibilities such as a D-D*bar molecule or c-cbar-gluon hybrid state.

In what follows, the masses of the X(3872) and X(3940) will be compared to the meson mass spectrum quantization patterns outlined in sections 1 and 2, and conclusions will be drawn only on a statistical basis, without any reference to models or theories and their predictions, nor to decay properties.

4. Analysis procedure

The only properties of the X(3872) and X(3940) mesons used in this analysis are their masses and respective errors (all masses will be quoted in MeV/c^2):

X(3872): m = 3872.0 ± 1.0

X(3940): m = 3940.0 ± 11.0 with dm = 68.0 ± 11.0

and the value of dm, in the vicinity of 70, indicates that the two states may actually be members of the same family, but the error on the mass of the X(3940) is too large to allow any discrimination.

The analysis procedure applied in [1] to each meson family, producing the results of figure 1b, consists of the following steps:

- compute mass numbers P_i: $m_i = P_i * u$ with a u-scan, with u varying in the range (33,38) and P_i the closest even integers, find the value of u corresponding to the best alignment on the basis of the R^2 correlation parameter, fit to compute u and its error;
- study a possible spin dependence by fitting separately the subsamples corresponding to different values of J, and comparing the results with a discrimination function;
- reject the value with the largest residual if it fulfils Chauvenet's criterion for the rejection of outliers;
- perform a weighted fit using the measurement errors, check for stability, compare with unweighted fit;
- evaluate the goodness-of-fit of the multiplicity hypothesis (p-value) by comparing the R^2 of the fit with the R^2 distribution of random samples of the same count in the same mass range.

In [1]the chi-squared of the weighted fit has not been used as a goodness-of-fit indicator, being in some cases too small, sign of an overestimation of errors, other times too big, possibly due to mass values that are marginally incorrect and do not trigger the outlier rejection criterion. In families with a large population across a wide mass range there are also signs of a slight modulation of u as a function of P that could result in high chi-squared values when some masses carry a very small error.

To match the X(3872) and X(3940) mesons with all the c-cbar families, and assess the likelihood that they might belong to one family or another, the same unweighted statistical procedures will be used, with differential reference to the chi-squared when appropriate, and also to the established u-grid symmetry rules.

Please refer to [1] for more details about the original analysis procedure, including the complete example of the pions, and the definition of relevant statistical variables.

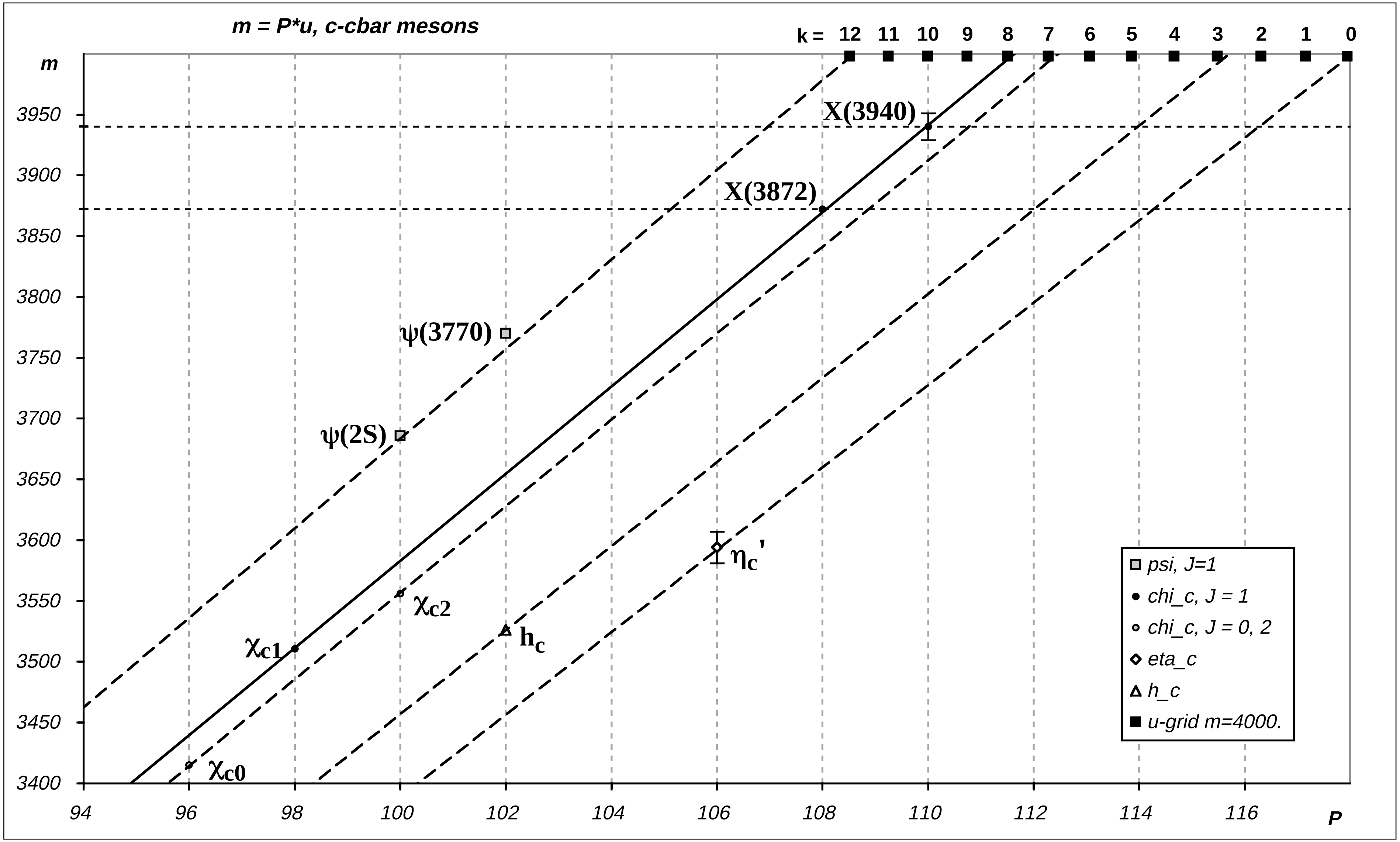


Fig. 3. Combined m=P*u plot for c-cbar meson families, and match with the X(3872) and X((3940). The skew lines are the result of the fits for the various families, the horizontal lines correspond to the masses of the two X mesons, and the vertical lines mark the even values of P. The 13 tacks along the upper right plot edge are the projection of the u-grid of figure 2 at m=4000. For most of the states the error bars on the mass are smaller than the size of the symbol. A good match corresponds to the combined intersection of an horizontal line with a skew line and a vertical line. The dm residuals are very large for the psi and chi_c(0,2) lines, appreciable for the eta_c, negligible for the chi_c(1) and the h_c. The h_c line hits the u-grid near k=3, while an even value would be expected. The chi_c(1) line hits the grid at k=8, one unit above the chi_c(0,2), corresponding to a du/dJ of 0.25 MeV/c^2, as expected by comparison with other families where u varies with the spin.

5. Comparison with the c-cbar meson families

5.1 psi (1^{--}) In the original analysis all 7 J/psi and psi states quoted by the PDG were considered, and exhibited a good alignment at u=36.8, but for the psi(4160) showing an almost-maximal residual of 32.8, which triggered a reject by Chauvenet's criterion (please see [1] for a discussion about the possible origin of the residual). All states are J=1, with the exception of the psi(3836) that is reported by the PDG as a possible J=2 according to charmonium considerations, but not really measured. By analogy with other c-cbar symmetric vector meson families such as the omega and the phi, it is expected that the psi mesons mass unit is spin dependent, so that a psi(2) at that mass would show a deviation from the psi(1) alignment of about 28 MeV/c^2, while its residual is 4.5. The fit and its quality remain the same with or without that state. J=1 can be predicted for the psi(3836), and the comparison sample is restricted to 5 states.

Against the line fit of the 5 psi(1) states, both the X(3872) and the X(3940) show a very large residual, close to the maximum. When added to the sample one at a time, both are rejected by Chauvenet's criterion. The goodness-of-fit p-value is downgraded from 0.96 to 0.81 for the X(3872), and to 0.74 for the X(3940).

Neither of the two states is a match for the psi(1) family.

5.2 eta_c (0^{-+}) The PDG lists two eta_c states, both with J=0, but the spin of the second state is just a quark model prediction. It does not really matter, as no du/dJ dependence is expected for this pseudoscalar family according to the u-grid rules. The measurement errors are small, 1.2 and 5.0, and the u-scan + fit procedure finds a very good alignment with residuals of -2.4 and 2, and the weighted fit has a chi-squared = 0.8 for 1 DOF.

The X(3872), when added to the sample, shows a residual of 8.9 and is rejected by Chauvenet. A combined u-scan with the two eta_c states and the X(3872) lowers the mass numbers of the two eta_c from P=88,106 to P=86,104, and the chi-squared of the weighted fit jumps to 15.5 for 2 DOF. The mass unit increases from 33.86 (k=0 on the u-grid) to 34.59 (k=3 on the u-grid), inconsistent with the u-grid rules. Keeping the original values of P for the eta_c, the weighted fit with the 3 states shows a chi-squared of 30.4 for 2 DOF.

Adding the X(3940) to the eta_c sample also triggers a Chauvenet's reject. The u-scan of the two eta_c mesons with the X(3940) also lowers the mass numbers of the eta_c states by 2 units, producing the same inconsistent shift on the value of u. The effect on the chi-squared of the weighted fit is not so dramatic, given the larger mass error of 11. MeV/c^2.

Neither state is compatible with the eta_c sample.

5.3 h_c (1^{+-}) Only one h_c state is known, with quantum numbers not defined. Assuming that it really is an h_c, by analogy with the h meson family and according to the u-grid rules no spin dependence is expected. For the scope of the present discrimination analysis, the low mass error of the h_c is a bonus.

A u-scan of the h_c with the X(3872) sets the mass numbers at P=102,112, corresponding to u=34.57, and the h_c with the X(3940) finds P=102,114 and the same value of u. Considering the three states together, the mass numbers are the same as with the scan two by two, and so is u. The chi-squared of the weighted fit with the three states is a very low 0.05 for 2 DOF, and the goodness-of-fit p-value computed with a montecarlo simulation is 0.99. Too good to be true? Actually the value of u for this triplet combination is 34.57, position k=3 on the u-grid, while the u-grid rules would predict an even value.

The two c-cbar X mesons are a very good match with the h_c from the point of view of the masses, but the resulting value of u looks problematic.

5.4 chi_c (0^{++}, 1^{++} , . .) The masses of the three known chi_c states, with spin = 0, 1 and 2, are known with an error smaller than 1 MeV/c^2, and being very close they provide stringent constraints. With m(2) - m(0) = 141.1 and m(1) - m(0) = 94.4, there is no way that the triplet can be part of the same 70 MeV/c^2-based multiplicity scheme, however the chi_c(0) and the chi_c(2) together show a very good alignment with the origin at P = (96, 100) and u = 35.57.

Assigning P=98 to the chi_c(1) gives a value of u = 35.82, so that u(1) - u(0) = 0.25, in line with the values of du/dJ seen in other families where u is spin dependent. But then how comes that the chi_c(2) and chi_c(0) are aligned so well for a value of u that checks with the u-grid rules? Either the value of u increases as expected going from J=0 to J=1, and then for J=2 flips back to the same value as J=0, or the spin of the chi_c(2) quoted by the PDG is not correct. With more accurate measurements and detection of more states time will tell.

Both the X(3872) and the X(3940), when added individually to the chi_c(0,2) sample, show very large deviations of about 25 MeV/c^2, and can be rejected by Chauvenet's criterion. Forcing them into a u-scan lowers P by 6 units in both cases and brings u out of the range of the u-grid and the goodness-of-fit p-values are only 0.83 and 0.86. There is no possible match.

Combining the two X mesons one by one with the chi_c(1) produces good matches with a consistent P= 98 for the chi_c(1) and very close values of u. The scan with three states aligns them at P=98, 108 and 110 with a goodness-of-fit p-value of 0.95 and u=35.84,. This corresponds to a du/dJ=0.27±0.02, in line with 0.23±0.04 of the omega, 0.29±0.1 of the phi and 0.25±0.07 of the a mesons.

The combination with the chi_c(1) is a satisfactory one, in terms of alignment, du/dJ and u-grid positioning.The results reported in this section can be appreciated graphically in the combined m vs P plot of figure 3.

6. Conclusions

The meson mass quantization patterns that have been used for the differential assignment of the X(3872) and X(3940) to the various c-cbar meson families are known with very good precision. Luckily the masses of most of the relevant c-cbar states carry an error that is small in comparison with a mass quantum, so that the statistical discrimination power is high. The u-grid symmetry rules and the selective dependence of u from the spin offer additional constraints, and in the end the only statistically significant and consistent match is with the chi_c(1).

This result is in agreement with the experimental constraints on the possible J^{PC} quantum numbers of the X(3872), recently published by the Belle Collaboration [10], where the accumulated evidence strongly favors a $J^{PC} = 1^{++}$ assignment for the X(3872). On the basis of the present results, the same quantum numbers are predicted for the X(3940). Moreover, Belle does not exclude 2^{++}, while this choice is rejected for both states by this mass analysis.

It will be interesting to see how those assignments will be integrated in the charmonium description. The belief in the quarkonium interpretation can be quite substantial, and is sometimes expressed in paradoxical terms or even by way of ipse dixits*,* for example: "Some theorists [..] remain hopeful that a c-cbar charmonium assignment can be found for the X(3872). To sort this all out, I think that the so-called hidden charm mesons can and will play a decisive role for reasons that include: the theory for these systems is well founded (and recently blessed by this year's Nobel Prize Committee) and has fewest ambiguities [..]" [11].

The results presented here are based on masses and their errors, without any reference to models or theories, and therefore do not have direct implications on aspects of the charmonium interpretation. It must be said however that the overall mass quantization patterns seem to be incompatible with some aspects of the strong sector of the standard model, and to disfavor the interpretation of higher-mass states as rotational excitations [12].

Without a support model, the predictions based on the meson mass quantization scheme can only be open-ended, stating that any new meson, if its mass is measured with sufficient accuracy and the quantum numbers are determined, must align with its fellow states with a statistically consistent residual. With precisely measured quantization steps of the order of 70 MeV/c^2, and errors on the mass of only a few units (and sometimes less than 1) the postdictive power can be substantial.

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Acknowledgements

I am very grateful to Simone Giani, Simone Gilardoni and Vladimir Ivancenko, who reviewed the draft and suggested various improvements.

Set in Helvetica and Symbol, formatted with PowerPoint and embedded Excel plots on an Apple Macintosh PowerBook Ti running MacOSX.