Seven at one blow: the mass system of the $\Theta^+$ baryons

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Abstract

Several $\Theta^+$ exotic baryon candidates have recently been identified using data from the JINR propane bubble chamber. The $pK_0^+$ invariant mass spectrum shows seven resonant structures ranging from 1487 to 1980 MeV/c$^2$, including the already established $\Theta(1540)^+$. In the present work the masses of the seven resonances are found to be equally spaced by about 70 MeV/c$^2$. This regularity is statistically relevant, and is compatible with an overall particle mass quantization scheme.

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In 2004 a group from JINR and Yerevan State University conducted a search for $S=1$ $\Theta^+$ exotic baryons by analyzing the data from the JINR 2m propane bubble chamber for the reaction $p+C_3H_8$ at 10 GeV/c [1]. In the $pK_0^+$ invariant mass spectrum, three resonant structures were clearly identified, with masses of 1540, 1613 and 1821 MeV/c$^2$ and widths $= 9.2, 16.1$ and 28 MeV/c$^2$ respectively. The first resonance is compatible with the $\Theta(1540)^+$ already detected by several other experiments, listed by the RPP of 2004 with three stars, and downgraded to two stars in 2005 [2]. Indications of further resonances with lower statistical significance were also present in the JINR data. In a more recent summary paper on exotic narrow resonance searches [3], P. Zh. Aslanyan, one of the authors of [1], reports a total of 7 $\Theta^+$ candidates. Their properties are summarized in table 1a.

<table>
<thead>
<tr>
<th>No.</th>
<th>$M_{\text{exp}} \pm \Delta M_{\text{exp}}$, MeV/c$^2$</th>
<th>$\Gamma_{\text{exp}} \pm \Delta \Gamma_{\text{exp}}$, MeV/c$^2$</th>
<th>$\Gamma \pm \Delta \Gamma$, MeV/c$^2$</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1487 ± 10</td>
<td>18.2 ± 2.1</td>
<td>9.2 ± 1.8</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>1540 ± 8</td>
<td>23.6 ± 6.0</td>
<td>16.1 ± 4.1</td>
<td>4.8</td>
</tr>
<tr>
<td>3</td>
<td>1613 ± 10</td>
<td>35.9 ± 12.0</td>
<td>28.90 ± 9.4</td>
<td>5.0</td>
</tr>
<tr>
<td>4</td>
<td>1690 ± 10</td>
<td>38.5 ± 12.0</td>
<td>31.0 ± 10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>5</td>
<td>1750 ± 10</td>
<td>39.0 ± 12.0</td>
<td>31.0 ± 10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>6</td>
<td>1821 ± 11</td>
<td>39.9 ± 12.0</td>
<td>31.0 ± 10.0</td>
<td>6.0</td>
</tr>
<tr>
<td>7</td>
<td>1980 ± 10</td>
<td>40.0 ± 12.0</td>
<td>31.0 ± 10.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

The concept that the masses of all particles might be multiples of a single mass unit was proposed by Y. Nambu [4] in 1952 on the basis of the few states known at that time, and extended by other authors (most notably by M. H. Mac Gregor [5]) as new states were discovered. A historical account can be found in [6]. Nambu’s conjecture was that one half of the $\pi$ mass, about 70 MeV/c$^2$, is a significant mass unit. On this basis he then attributed to all known particles a “mass number”, integer or half-odd: 0 for the photon, 3/2 for the $\mu$, 2 for the $\pi$, 7 for the K, 27/2 for the nucleon and so on. He noticed also that: the mass unit agrees with Heisenberg’s natural unit $m_e/\alpha (= 70.02$ MeV/c$^2$); bosons have integer, while fermions half-integer, mass number; the electron mass and the $\pi^-$ mass difference correspond to a kind of fine structure.

Being the $\Theta^+$ masses spaced by about 70 MeV/c$^2$, it is interesting to compare them with this quantization scheme. This kind of comparative analysis has recently been used by the present author to identify Belle’s $X$ mesons [7] and scrutinize a group of $\phi^0$ meson candidates [8].

The quantization with a single unit is statistically relevant only below 1 GeV/c$^2$, and above that value it just blurs out. As noticed by M. H. Mac Gregor, and systematized by the present author for all the mesons [6], the rule is valid throughout the mass spectrum, with mass units that are function of the particle family defined by the quark composition and $J^{PC}$.
4. The meson and baryon mass systems

To avoid half-integer mass numbers, the mass unit \( u \) used here and in the references by the present author is half the one of Nambu and Mac Gregor, \( 35 \text{ MeV}/c^2 \), thus \( m_i = u j \cdot P \), with integer mass numbers \( P \), even for mesons and odd for baryons, and \( u_j \) a type-dependent mass unit. The meson results are already published [6] and show that: the quantization hypothesis is statistically relevant; some families show a variation of \( u \) with the spin \( du/dJ \) of about 0.25 MeV/c\(^2\); the mass units \( u_j \) for the various families are themselves quantized on a \( u \)-grid of 13 values between 33.88 and 36.84 MeV/c\(^2\), spaced by about 0.25 MeV/c\(^2\), with the central value at 35.36 MeV/c\(^2\). In addition, their location on the \( u \)-grid shows intriguing correlations with the quantum numbers (see figure 2a).

The baryon results are almost final and will be published shortly [9]. Their mass quantization is also bi-linear and similar to the mesons, with features that are anticipated here in view of the \( \Theta^+ \) comparison: the mass quantization hypothesis is statistically relevant, in some cases with a \( du/dJ \) dependence of about 0.25 MeV/c\(^2\), but the range of the mass units is restricted to the 7 innermost locations of the \( u \)-grid, from 3 to 9. Moreover, the mass units of the \( N \) and \( \Delta \) families are located below the middle point, strange families above, charmed below, as indicated by the ranges above the horizontal axis in figure 2a.

5. The \( \Theta^+ \) baryon candidates in context

The mass numbers \( P_i \): \( m = u \cdot P \), are derived with a scan, varying \( u \) in an interval around 35, computing the corresponding odd integers \( P \), from \( u \) and the masses, and choosing the set of values that maximizes the \( R^2 \) correlation coefficient (= 1 for a perfect alignment). A linear LS fit constrained at \((0,0)\) finds \( u = 35.89 \pm 0.07 \), with a \( \chi^2 \) of 4.5 for 6 D.O.F. The spread of the residuals is consistent with a homogeneous sample, no spin dependence and no outliers.

The resulting mass numbers are listed in the table at figure 2b, and used as abscissae in the plot. The statistical relevance is expressed by the \( p \)-value (sometimes referred to as confidence level) of the null hypothesis: 7 mass values randomly distributed in the same interval. The \( R^2 \) of the fit = 0.9973 is compared with the \( R^2 \) distribution of Montecarlo-generated random samples, and the resulting \( p \)-value is 2.3%, the probability that a random sample is as correlated as the experimental sample or better. Details of the procedure and definitions of statistical variables can be found in [6].

6. Conclusions

The masses of the 7 \( \Theta^+ \) candidates are consistent with a family of baryons of homogeneous quark composition and \( J^{PC} \). In addition, either all the states have the same spin, or in this family the mass unit is not spin dependent. The value of their mass unit is at position 8 in the \( u \)-grid, in the range of the established strange baryon families. From this viewpoint, the \( \Theta^+(1540)^+ \), scrutinized with its fellow states, deserves at least to get back its 3rd star.

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Fig. 2. \( \Theta^+ \) candidates: 2a, mass units of all meson families with ranges for baryons, adapted from [6]; 2b, \( m = u \cdot P \) plot for the 7 \( \Theta^+ \) states, linear fit and table with residuals.
Bibliography


Acknowledgements
I am grateful to Petros Aslanyan for clarifying aspects of the data and their interpretation, to Simone Giani for reviewing this paper, and to Malcolm Mac Gregor for his interest in this work.

Revision Record
11-AUG-2005 preprint, original electronic submission